
Constitutive Norms and Counts-as Conditionals

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ABSTRACT. The chapter introduces the theory of constitutive rules and counts-as statements from a philosophical/informal point of view and addresses existing attempts to provide a formalization of it. These attempts are concisely described and compared along three main lines: one pertaining to their contribution to the clarification of a set of selected benchmark problems (e.g., institutional power, classificatory rules, conventions etc.); the second pertaining to their methodology (axiomatic/syntactic vs. model-theoretic/semantic approaches); the third pertaining to their strictly formal properties. On the grounds of such systematic comparisons the chapter also identifies open questions and points to future research directions that the authors consider essential in order to shed further light on constitutive rules and counts-as.

1 Introduction

Constitutive norms—or rules¹—are a commonplace of social reality as we know it. They make possible basic ‘institutional’ actions such as the making of contracts, the issuing of fines, the decreeing of divorces. With the work of Searle (in particular [Searle, 1969] and [Searle, 1995]), to which we will often return in the chapter, these norms have acquired a somewhat canonical form, the one of *counts-as conditionals*:

$$X \text{ counts as } Y \text{ in context } C.$$

This canonical presentation of constitutive norms paved the way for the natural question of what the logic of these rules is, in terms the logic of counts-as conditionals. The present chapter reviews the attempts that have been made at understanding this logic since the first paper on the issue was published in 1996 [Jones and Sergot, 1996]. As we will see, these investigations have given rise to a lively inter-disciplinary research field which has produced a rich and varied landscape of logical systems.

¹We use the two terms interchangeably in the chapter

Outline of the chapter.

The chapter will develop along the following lines. Section 2 provides an overview of that philosophical work on constitutive norms and counts-as conditionals from which later formal work has developed, and which has inspired all the different formal approaches which will constitute the core of the chapter. Those approaches are dealt with in Section 3. That section provides a bird's eye view of the landscape of formal accounts of constitutive norms and counts-as conditionals and sets the ground for their analysis and comparison, which is developed in Section 5. We consider this latter section as the main contribution of this chapter, where the various approaches to counts-as are classified and compared with respect to three key criteria: one, the aspects of constitutive rules they aim at accounting for formally; two, the methods they use in their formal analyses; three, the formal properties of the different formalizations. Finally, Section 6 deals with some of the most challenging—in the authors' view—open problems in the field. Section 7 briefly recapitulates and concludes the chapter.

2 Theory of counts-as—informal contributions

The concepts of counts-as conditional and constitutive rule have been informally discussed, under different names, in several philosophical sub-disciplines such as: the theory of institutions [Searle, 1969; Cherry, 1973; Searle, 1995], the theory of action [Goldman, 1976], the theory of norms [Von Wright, 1963; Alchourrón and Bulygin, 1971], the theory of law [Rawls, 1955; Ross, 1957; Peczenik, 1989; Bulygin, 1992], the theory of communication [Searle, 1969; Jones and Parent, 2004; Fornara *et al.*, 2007].

The present section is devoted to a brief discussion of the main features of counts-as and constitutive rules as they emerge from some of the philosophical literature just mentioned. This has to be intended as a non-exhaustive overview, emphasizing some of the aspects which have obtained particular attention by the formal approaches to counts-as that will be discussed later.² These aspects are: the opposition between regulative and constitutive norms; the opposition between brute and institutional facts and the contextual nature of the latter; the classificatory and definitional role played by constitutive norms; finally, their use as a basic technique of presentation of the law.

²There are of course also other philosophical works that address these topics, but we have chosen to focus on those that have perhaps most influenced the development of formal-logical theories of counts-as conditionals.

2.1 Constitutive vs. regulative norms

Regulative norms are what most commonly go simply under the name ‘norm’. They have deontic content and they indicate what is obligatory, permitted, forbidden. A very much emphasised feature of constitutive rules is that they do not regulate actions or states-of-affairs, but rather they define new possible actions or states of affairs.

The distinction is very explicitly stated in Searle and Bulygin, as the following quotes illustrate:

“As a start, we might say that regulative rules regulate antecedently or independently existing forms of behavior [...]. But constitutive rules do not merely regulate, they create or define new forms of behavior” [Searle, 1969, p. 33].

“Where the rule is purely regulative, behaviour which is in accordance with the rule could be given the same description or specification (the same answer to the question “What did he do?”) whether or not the rule existed, provided the description or specification makes no explicit reference to the rule. But where the rule (or system of rules) is constitutive, behaviour which is in accordance with the rule can receive specifications or descriptions which it could not receive if the rule did not exist” [Searle, 1969, p. 35].

“If we do not comply with such rules [constitutive rules], the result is not a sanction or a punishment, for it is not breach or violation of any obligation, nor an offence, but nullity [Bulygin, 1992, p. 208].

Although the difference between regulation and constitution might be clear, it is much less clear what the notion of constitution precisely amounts to. In the philosophical literature, a common way to describe the notion of constitution is by interpreting it as the fact that the very existence of constitutive norms is a necessary condition for the existence of certain social practices like games, such as baseball or chess:

“In the case of actions specified by practices it is logically impossible to perform them outside the stage-setting provided by those practices, for unless there is the practice, and unless the requisite proprieties are fulfilled, whatever one does, whatever movements one makes, will fail to count as a form of action which the practice specifies. What one does will be described in some other way.

One may illustrate this point from the game of baseball. Many of the actions one performs in a game of baseball one can do by oneself or with others whether there is the game or not. For example, one can throw a ball, run, or swing a peculiarly shaped piece of wood. But one cannot steal base, or strike out, or draw a walk, or make an error, or balk; although one can do certain things which appear to resemble these actions such as sliding into a bag, missing a grounder and so on. Striking out, stealing a base, balking, etc., are all actions which can only happen in a game. No matter what a person did, what he did would not be described as stealing a base or striking out or drawing a walk unless he could also be described as playing baseball, and for him to be doing this presupposes the rule-like practice which constitutes the game” [Rawls, 1955, p. 25]

Or, similarly, as Searle puts it about the game of chess:

“[W]hat the ‘rule’ seems to offer is part of a definition of ‘checkmate’ [...] That, for example, a checkmate in chess is achieved in such and such way can appear now as a rule, now as an analytic truth based on the meaning of ‘checkmate in chess’. That such statements can be construed as analytic is a clue to the fact that the rule in question is a constitutive one. The rules for checkmate [...] must ‘define’ *checkmate in chess* [...] in the same way that [...] the rules of chess define ‘chess’ [...]” [Searle, 1969, p. 34].

Let us elaborate these observations by means of a concrete example of the simple type of constitutive norms consisting of the rules of chess .

EXAMPLE 1.1 (Checkmate) *The following are constitutive rules of the game of chess: a checkmate occurs if a king is under direct attack and all of its moves lead to a position which is also under direct attack; a piece is under direct attack if an opponent’s piece has an available move to its square; an available move of a piece is a move according to the piece’s codified style of moving. These rules describe a class of situations on the chessboard, all those situations in which a checkmate occurs. Figure 1.1 depicts a simple instance of that class.*

2.2 Brute vs. institutional facts, and contextual nature of constitutive norms

To continue with Example 1.1, the situation illustrated in Figure 1.1 can be crudely described by giving all the pieces’ coordinates on the chessboard.

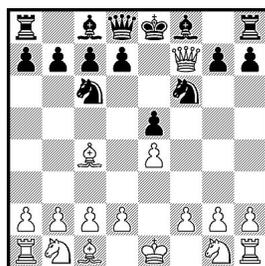


Figure 1.1. A configuration on a chess board instantiating a situation, viz. the fact that the black king cannot move, which *counts as* a checkmate (of black).

In virtue of the rules of chess, that configuration of pieces is such that the black king cannot move according to its style of moving, and hence it is checkmated. Searle would call the description given in terms of coordinates a ‘brute fact’, and checkmate an ‘institutional fact’. The link between the two is granted by the rules of chess.

In other words, the way constitutive norms define new forms of actions or new states-of-affairs is by relating them to something already existing or established. So, constitutive norms may relate “brute facts” [Anscombe, 1958] to “institutional facts”.³ A precursor of the distinction brute/institutional is the German legal philosopher Pufendorf who, already in the 17th century, made a similar distinction between physical and moral entities:

“Now, as the original manner of producing physical entities is creation, there is hardly a better way to describe the production of moral entities than by the word ‘imposition’ [*impositio*]. For moral entities [*entia moralia*]⁴ do not arise from the intrinsic substantial principles of things but are superadded to things already existent and physically complete [read brute facts]” [Pufendorf, 1688, p. 100-101].

This distinction, however, plays a central role in Searle’s theory who also stresses a further aspect of it, namely the contextual nature of the ‘constitution’. Context has been incorporated by Searle as an explicit component of what we called above counts-as conditionals:

³As we shall see later, they may also relate institutional facts to other institutional facts.

⁴Cf. [Ricciardi, 1997].

[...] “institutions” are systems of constitutive rules. Every institutional fact is underlain by a (system of) rule(s) of the form “X counts as Y in context C” [Searle, 1969, pp. 51-52].

The contextual nature of constitutive norms is not obvious in examples inspired by game-playing such as Example 1.1. It becomes instead evident when talking about social practices such as, for instance, marriage.

EXAMPLE 1.2 (The bringing about of a marriage) *“By the power vested in me by the State c, I now pronounce you husband and wife”.* The declaration makes explicit the context *c* in which the new state-of-affairs occurs, and that this state occurs as result of the declaration itself. The power to which the declaration refers is rooted in a rule of State *c*, stating that such a declaration in the wedding ritual, counts as the creation of a state-of-affairs in which the couple is married.

2.3 Constitutive rules as a “technique of presentation”

We conclude the section by briefly reporting on work by Ross [Ross, 1957]. This work offers a quite illuminating view of constitutive rules which focuses on the ‘*raison d’être*’ of such rules within legal systems. Constitutive rules seem to be a pervasive feature of legal systems, but why is it so? Or, said otherwise, what are constitutive rules actually good for?

Ross provides an answer to the question by reporting a lively story, of which we quote an excerpt:

“On the Nosulli Islands in the South Pacific lives the Noît-cif tribe, generally regarded as one of the more primitive peoples to be found in the world today [...]. This tribe [...] holds the belief that in the case of an infringement of certain taboos—for example, if a man encounters his mother-in-law, or if a totem animal is killed, or if someone has eaten of the food prepared for the chief—there arises what is called *tû-tû*. The members of the tribe also say that the person who committed the infringement has become *tû-tû*. It is very difficult to explain what is meant by this. [...] *tû-tû* is conceived as a kind of dangerous force [...] a person who has become *tû-tû* must be subjected to a special ceremony of purification” [Ross, 1957, p. 812].

In Ross’s view a term such as *tû-tû* is a word devoid of any meaning, it is a term without reference.⁵ Nonetheless, terms of this type do play a key

⁵We do not endorse this view here, as it is actually a matter of controversy in philosophy (cf. [Hindriks, 2009]), nor do we intend to dig deeper into the issue of reference and denotation of *tû-tû*-like terms. What matters for our purposes here is Ross’s analysis of the function of such terms within a normative system.

role in the specification of norms. They are the bridge—in logical terms the interpolant—which enables inferences connecting concrete facts, to normative consequences. To use Ross’s example:

- (i) If a person has eaten of the chief’s food she is *tû-tû*.
- (ii) If a person is *tû-tû* she has to be subjected to a ceremony of purification.
- (iii) If a person has eaten of the chief’s food she has to be subjected to a ceremony of purification.

To say it in the manner of Searle, *tû-tû* is an institutional fact and Statement (i) connects it to a specific brute fact. In the counts-as terminology: the fact that a person has eaten of the chief’s food counts, in the context of Noît-cif tribal laws, as the fact that she is *tû-tû*. Statement (ii) then introduces a normative consequence linked to the institutional fact ‘*tû-tû*’, stating what the effects of being *tû-tû* are (in this case normative effects). Taken together, they allow the inference of Statement (iii), where *tû-tû* does not occur any more, and which makes the connection between the fact at issue and its normative consequences explicit.

Ross stresses the analogy of the above inference pattern to the sort of rulings we are bound to encounter in modern legal codes.

“We find the following phrases, for example, in legal language:

- (1) If a loan is granted, there comes into being a claim;
- (2) If a claim exists, then payment shall be made on the day it falls due;

This is only a roundabout way of saying:

- (3) If a loan is granted, then payment shall be made on the day it falls due.

The claim mentioned in (1) and (2), but not in (3), is obviously, like *tû-tû*, not a real thing; it is nothing at all, merely a word, an empty word devoid of any semantic reference” [Ross, 1957, p. 817–818].

The point is thus made that “our legal rules are in a wide measure couched in a ‘*tû-tû*’ terminology” [Ross, 1957, p. 817]. Terms such as claim, right, duty, ownership work exactly like *tû-tû* allowing us to connect a set of concrete circumstances to a set of legal or, more generally, normative consequences. This detour via *tû-tû*-terms might be dispensed with, but the price to pay

$F_1 - C_1$	$F_2 - C_1$	\dots	$F_n - C_1$
$F_1 - C_2$	$F_2 - C_2$	\dots	$F_n - C_2$
\vdots	\vdots		\vdots
\vdots	\vdots		$F_{n-1} - C_{m-1}$
$F_1 - C_m$	$F_2 - C_m$	\dots	$F_n - C_m$

Figure 1.2. $n \cdot m$ rules connecting n (brute) facts to m (normative) consequences.

is a rather cumbersome formulation. To realize this, suppose you were asked to connect, by means of rules, each of n (brute) facts F_1, \dots, F_n to m (normative) consequences C_1, \dots, C_m . The naive way to do that would consist in connecting each fact to each consequence, thereby producing $n \cdot m$ different rules of the form $F_i - C_j$, with $1 \leq i \leq n$ and $1 \leq j \leq m$. This solution is displayed in Figure 1.2.

But now observe what the addition of a $\hat{t}\hat{u}$ - $\hat{t}\hat{u}$ -like term—let us call it Y —would allow you to do. The same situation would be expressible via $n + m$ rules: n rules of the form $F_i - Y$ connecting each brute fact to the term Y , and m rules of the form $Y - C_j$, with $1 \leq j \leq m$, connecting Y to each normative consequence. This solution is displayed in Figure 1.3.⁶ Each of the $F_i - Y$ rules can be consistently thought of as a counts-as statement of the form F_i counts as Y , in the context of the thereby defined normative system or institution.

Ross's point is precisely that $\hat{t}\hat{u}$ - $\hat{t}\hat{u}$ -like terms or, in Searlean terminology, institutional facts, enable a very manageable and effective “technique of presentation” [Ross, 1957, p. 821] for systems of norms. And within this picture constitutive rules play therefore a central role and have to be considered as a basic building block for the construction of normative systems.

3 Theory of counts-as—formal contributions

The thrust to the development of a formal analysis of constitutive rules could be traced back to Searle's work itself where, in both [Searle, 1969] and [Searle, 1995], constitutive rules are constantly related to a specific syntactic

⁶The interested reader might already glance over the formalization of Figure 1.3 provided later on within Section 3 in Formula 1.6.

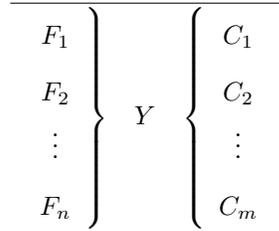


Figure 1.3. $n + m$ rules connecting n (brute) facts to m (normative) consequences.

form: “Constitutive rules have the form: ‘X counts as Y in context C’”.⁷ Once a special syntactic form is in focus, the natural question arises as to what the logic of that form is. Spanning across several techniques and methods, this section summarizes the findings of those authors that took up the quest for a logic of statements of the form ‘X counts as Y in context C’.

We can identify five main groups of contributions to the formal analysis of counts-as and constitutive norms: by Jones *et al.* [Jones and Sergot, 1996; Jones and Parent, 2004; Jones and Parent, 2007]; by Gelati *et al.* [Gelati *et al.*, 2002; Artosi *et al.*, 2004; Gelati *et al.*, 2004]; by Boella *et al.* [Boella and van der Torre, 2003; Boella and Van der Torre, 2004; Boella and van der Torre, 2006]; by Lindahl *et al.* [Lindahl and Odelstad, 2006; Lindahl and Odelstad, 2008a; Lindahl and Odelstad, 2008b]; by Grossi *et al.* [Grossi *et al.*, 2005; Grossi, 2007; Grossi *et al.*, 2006a; Grossi *et al.*, 2006b; Grossi *et al.*, 2008; Grossi, 2008; Grossi, 2010]; by Lorini *et al.* [Gaudou *et al.*, 2008; Lorini and Longin, 2008; Lorini *et al.*, 2009]; and by Governatori *et al.* [Governatori and Rotolo, 2008].

The present section describes the key features of each of these approaches and offers a concise overview of the different techniques used up till now in the formal analysis of counts-as. We will refrain from providing full details about each of the approaches, and will rather focus on their key definitions, trying to emphasize the characteristic features of each of them. This will give us a solid basis on which to ground the systematic comparison of these approaches to be articulated in Section 5. The section divides the formalisms into two groups: the ones based on modal logic, and the ones

⁷However, Searle was certainly not the only—nor indeed the first—philosopher in the modern period to describe constitutive rules in terms of the ordinary English verb ‘count as’. See, e.g., [Rawls, 1955, p. 25].

based on alternative formalisms.

Modal logics of counts-as

To this group belong the works by Jones *et al.* , Gelati *et al.* , Grossi *et al.* and Lorini *et al.* .

3.1 Jones *et al.*

The formal analysis of constitutive norms and counts-as conditionals starts with [Jones and Sergot, 1996]. This work is concerned with isolating a core of logical principles governing the use of statements of the form “X counts as Y in context (or, institution) c ”, which are given a representation in a logical language by means of a connective \Rightarrow_c . Counts-as conditionals are thus represented by formulae of the type $\varphi_1 \Rightarrow_c \varphi_2$, and are studied within the framework of conditional logics in the Chellas tradition [Chellas, 1980].

The resulting conditional logic of counts-as is taken to validate, on top of propositional logic, the following principles:

$$\varphi_2 \leftrightarrow \varphi_3 / (\varphi_1 \Rightarrow_c \varphi_2) \leftrightarrow (\varphi_1 \Rightarrow_c \varphi_3) \quad (1.1)$$

$$\varphi_1 \leftrightarrow \varphi_3 / (\varphi_1 \Rightarrow_c \varphi_2) \leftrightarrow (\varphi_3 \Rightarrow_c \varphi_2) \quad (1.2)$$

$$((\varphi_1 \Rightarrow_c \varphi_2) \wedge (\varphi_1 \Rightarrow_c \varphi_3)) \rightarrow (\varphi_1 \Rightarrow_c (\varphi_2 \wedge \varphi_3)) \quad (1.3)$$

$$((\varphi_1 \Rightarrow_c \varphi_2) \wedge (\varphi_3 \Rightarrow_c \varphi_2)) \rightarrow ((\varphi_1 \vee \varphi_3) \Rightarrow_c \varphi_2) \quad (1.4)$$

$$(\varphi_1 \Rightarrow_c \varphi_2 \wedge \varphi_2 \Rightarrow_c \varphi_3) \rightarrow (\varphi_1 \Rightarrow_c \varphi_3) \quad (1.5)$$

The inference rules in Formulae 1.1 (right logical equivalence) and 1.2 (left logical equivalence) simply state the rather uncontroversial property that counts-as conditionals are closed under substitution of provable equivalents in the antecedent as well as the consequent.

Formulae 1.3 and 1.4 express the properties of conjunction of the consequent and, respectively, disjunction of the antecedent, both to be considered quite natural for statements of the type “X counts as Y in c ”. In fact they allow for a natural rendering of the logical content of the “technique of presentation” enabled by counts-as statements, which we briefly discussed in Section 2.3. We could recast the content of Figure 1.3 by means of operator \Rightarrow_c in the following way:

$$\bigvee_{1 \leq i \leq n} F_i \Rightarrow_c Y \quad \wedge \quad Y \Rightarrow_c \bigwedge_{1 \leq j \leq m} C_j \quad (1.6)$$

assuming consequences C_j to be expressed as institutional facts too. In other words, Formulae 1.3 and 1.4 allow one to cluster n counts-as statements of the form $F_i \Rightarrow_c Y$ connecting n brute facts F_i to the institutional fact Y ,

and m similar statements of the form $Y \Rightarrow_c C_j$ from the institutional fact Y to m institutional facts C_j .

Notice that not quite as intuitive and natural are the converses of Formulae 1.3 and 1.4, and in particular the converse of Formula 1.4 (antecedent strengthening⁸), for the reason perspicuously illustrated by the following quote:

“The point is essentially this: suppose that x is empowered to marry couple a and b by performing ritual R . Now suppose that some other agent y brings it about that x performs ritual R — y , let us imagine, successfully exercises influence over x by some means or other. So x performs the ritual and the couple a and b are married. Despite his successful exercise of influence, we would not here want to say that y too was empowered, by institution s , to marry the couple. Institutional power is not *transferable* in that way.” [Jones and Sergot, 1996, p. 434]

Finally, Formula 1.5 expresses the transitivity of counts-as statements, which [Jones and Sergot, 1996] accepts on the grounds of the property of *conventional generation* as studied in [Goldman, 1976].

As we will see later in Section 5, Formulae 1.1-1.5 constitute a ‘minimal core’ of logical principles for the logic of counts-as and have hardly been criticized (except for the more controversial property of transitivity, Formula 1.5) by later proposals for the formal analysis of counts-as.

In addition to the above principles, [Jones and Sergot, 1996] links the logic of counts-as conditionals to the logic of a modality D_c . The intuitive reading of a formula $D_c\varphi$ is “it is a constraint of institution c that φ ”. The logic of D_c is taken to be that of the normal modal system **KD**⁹ and it is linked to the logic of counts-as conditionals by the following schemata:

$$(\varphi_1 \Rightarrow_c \varphi_2) \rightarrow D_c(\varphi_1 \rightarrow \varphi_2) \tag{1.7}$$

$$(\varphi_1 \Rightarrow_c \varphi_2) \rightarrow (\varphi_1 \rightarrow D_c\varphi_1). \tag{1.8}$$

⁸In the current notation such a property is formulated as follows:

$$\varphi_1 \Rightarrow \varphi_2 \rightarrow \varphi_1 \wedge \varphi_3 \Rightarrow \varphi_2.$$

⁹That is, the modal logic with modality D_c containing the axioms of propositional logic, axioms

$$\begin{array}{ll} \mathbf{K} & D_c(\varphi_1 \rightarrow \varphi_2) \rightarrow (D_c\varphi_1 \rightarrow D_c\varphi_2) \\ \mathbf{D} & \neg D_c\perp \end{array}$$

and the rules of modus ponens (RM) and necessitation (RN).

Formula 1.7 states, intuitively, that counts-as conditionals of a given institution c are a subset of the constraints operative in institution c . The second one, Formula 1.8, states that, if a state-of-affairs occurs as an antecedent in a counts-as conditional, then, if that state-of-affairs is the case it is also “institutionally” the case, that is, it is recognized by the institution concerned.

The rationale for the choice of schemata such as Formulae 1.7 and 1.8, is motivated in [Jones and Sergot, 1996] by the attempt to provide a systematization of inference patterns that arise in the common use of counts-as statements. The sort of logic arising from the interaction of Formulae 1.1-1.5 with the logic of the D_c modality allows for specific reasoning patterns concerning counts-as to be given a formal systematization. In particular, the analysis proposed in [Jones and Sergot, 1996], besides being philosophically informed by contributions such as those of Bulygin, Goldman, Rawls and Searle, among others, aims precisely at accounting for the following sort of inference.

EXAMPLE 1.3 (Institutional detachment) *Let us provide a first formalization of Example 1.2. The counts-as rule at issue can be expressed as $p \Rightarrow_c m$. Roughly, the state-of-affairs in which the officer pronounces the couple husband and wife (p in symbols), in the context of institution c , counts as the couple being married (m in symbols). Now, by assuming that p is the case, we would like to be able to infer that m actually holds in the context of institution c . This reasoning pattern is sound in the Jones et al.’s logic:*

$$\{p \Rightarrow_c m, p\} \vdash D_c m \quad (1.9)$$

To prove this rule, from $p \Rightarrow_c m$, Formula 1.8 and modus ponens we can infer $(p \rightarrow D_c p)$, from which, by p , modal principles and modus ponens we can conclude $D_c m$.

Besides triggering essentially all the further proposals on the formalization of counts-as statements, the framework developed in [Jones and Sergot, 1996] has been applied to the theory of signaling conventions in [Jones and Parent, 2004; Jones and Parent, 2007].

3.2 Gelati *et al.*

Gelati *et al.* [Gelati *et al.*, 2002; Gelati *et al.*, 2004] define a counts-as operator \Rightarrow_c in terms of a more basic form of conditional \Rightarrow , which they call *normative*, and, following in the footsteps of [Jones and Sergot, 1996], in terms of a D_c modality:

$$\varphi_1 \Rightarrow_c \varphi_2 \quad := \quad (\varphi_1 \Rightarrow D_c \varphi_2) \wedge (D_c \varphi_1 \Rightarrow D_c \varphi_2) \quad (1.10)$$

The rationale of this definition, the authors claim, resides in interpreting statements of the type “ X counts as Y in context c ” as statements asserting that “from X follows that Y is institutionally the case in c and that from the fact that X is institutionally the case in c it also follows that Y is institutionally the case in c ”. To capture that intuitive reading, and thereby substantiate the definition in Formula 1.10, the authors choose non-normal logics for both the \Rightarrow_c and the D_c operators:

- The \Rightarrow operator is taken to correspond to cumulative reasoning¹⁰ together with the rather non-standard inference rule inspired by non-monotonic reasoning:

$$\frac{\Phi \vdash \varphi_1 \quad \Phi \vdash \varphi_1 \Rightarrow_c \varphi_2}{\Phi \vdash \varphi_2}$$

if for any φ'_1 such that $\Phi \vdash \varphi'_1 \rightarrow \varphi_1$, $\Phi \not\vdash \varphi'_1 \Rightarrow_c \neg\varphi_2$, and where Φ is a set of formulae.

- The D_c operator is taken to be a non-normal modality obeying only the two principles: $D_c\varphi \rightarrow \neg D_c\neg\varphi$ and $(D_c\varphi_1 \wedge D_c\varphi_2) \rightarrow D_c(\varphi_1 \wedge D_c\varphi_2)$.

The structural properties of \Rightarrow_c arising from Formula 1.10 are not explored and no metalogical results (e.g., soundness, completeness) are investigated for the proposed logic of counts-as.

3.3 Grossi *et al.*

In a series of papers starting with [Grossi *et al.*, 2005] Grossi *et al.* develop a theory of counts-as conditionals within the framework of normal modal logic (i.e., of extensions of the modal system **K**). The upshot of their analysis consists in isolating a family of different operators, in ascending logical strength, all capturing different semantic components which seem to be involved in statements of the type “ X counts as Y in context c ”. Four notions of counts-as are studied, which are informally presented in the following list:

Classificatory counts-as: situations of the type X are all of the type Y in context c . This interpretation considers counts-as conditionals simply as contextual classifications and is rooted in a simple observation made in [Jones and Sergot, 1996]:

¹⁰To be precise, in [Gelati *et al.*, 2002] it is argued that the logic of counts-as corresponds to preferential reasoning, while in [Gelati *et al.*, 2004] it is considered to correspond *at least* to cumulative reasoning, i.e., preferential reasoning without the property of disjunction of the antecedents (see [Kraus *et al.*, 1990]).

“There are usually constraints within any institution according to which certain states of affairs of a given type count as, or *are to be classified as*, states of affairs of another given type” [Jones and Sergot, 1996, p. 139].

Proper classificatory counts-as: situations of the type X are all of the type Y in context c , and this does not hold in general. This interpretation builds on the previous one requiring that the statement “situations of the type X are all of the type Y ” be not a universal truth or, in other words, that it be something proper of context c .

Ascriptive counts-as: situations of the type X are all of the type Y in context c , and type Y is a newly introduced concept. This interpretation also builds on the first one, and makes explicit that what happens in a counts-as statement, besides the classificatory content, is also the ‘creation’ of a new concept, without which the classification would not be possible.

Constitutive counts-as: situations of the type X are all of the type Y in context c , and this does not hold in general and, in addition, context c is explicitly defined by a set Γ of statements including that X implies Y . Finally, this last interpretation takes into consideration that:

“Rules are constitutive if and only if they are part of a set of rules. Strictly speaking, there is no such thing as *a* rule that is constitutive in isolation” [Ricciardi, 1997, p. 5]

This means that a counts-as statement, besides its classificatory content, is also always part of a set of rules which, together, define the context c of the statement.¹¹

The fundamental component of the formalization of all these different notions is, essentially, a simple modal logic of contexts where contexts are represented as modal operators $[c]$ with the following interpretation:

$$\mathcal{M}, s \models [c]\varphi \iff S_c \subseteq \|\varphi\|_{\mathcal{M}} \quad (1.11)$$

where $\|\varphi\|_{\mathcal{M}}$ denotes the truth-set of φ in \mathcal{M} and S_c is the set of states corresponding to context c .¹² That is, φ holds in context c if and only if

¹¹Note that this is in line with the warning raised in [Makinson, 1999]: “no logic of norms without attention to a system of which they form part”.

¹²For a thorough exposition we refer the reader to, e.g., [Grossi, 2007].

all states in c are φ states.¹³ This is all that is needed to express the classificatory core meaning of counts-as statements, the so-called *classificatory counts-as*:

$$\varphi_1 \Rightarrow_c^{cl} \varphi_2 := [c](\varphi_1 \rightarrow \varphi_2) \quad (1.12)$$

On the top of this simple definition, Grossi *et al.* then investigate a number of normal modal operators which, in interaction with the context modality, allow one to capture all the aforementioned different interpretations of counts-as statements. Here we will only report the semantics of such operators, and show how they are used to obtain the desired formalizations. Such operators are the well-known universal modality $[U]$ (cf. [Blackburn *et al.*, 2001]), a linguistic release operator $\Delta_{\sigma(\varphi)}$ borrowed from [Krabbendam and Meyer, 2000; Krabbendam and Meyer, 2003], and a contextual complement operator $[-c]$ introduced in [Grossi *et al.*, 2006b; Grossi *et al.*, 2008]:

$$\mathcal{M}, s \models [U]\varphi \iff S \subseteq \|\varphi\|_{\mathcal{M}} \quad (1.13)$$

$$\mathcal{M}, s \models \Delta_{\sigma(\varphi_2)}\varphi_1 \iff \forall s' \text{ s.t. } s' \sim_{\sigma(\varphi_2)} s : \mathcal{M}, s' \models \varphi_1 \quad (1.14)$$

$$\mathcal{M}, s \models [-c]\varphi \iff S \setminus S_c \subseteq \|\varphi\|_{\mathcal{M}} \quad (1.15)$$

where $\sigma(\varphi_2) \subseteq \mathbf{P}$ denotes the vocabulary of formula φ_2 and the relation $\sim_{\sigma(\varphi_2)}$ is an indistinguishability relation holding between states which are equivalent with respect to the atoms in the vocabulary $\sigma(\varphi_2)$ (cf. also [Grossi, 2009]). These operators support the following extensions of the definition of classificatory counts-as given in Formula 1.12:

$$\varphi_1 \Rightarrow_c^{cl+} \varphi_2 := [c](\varphi_1 \rightarrow \varphi_2) \wedge \neg[U](\varphi_1 \rightarrow \varphi_2) \quad (1.16)$$

$$\varphi_1 \Rightarrow_c^{As} \varphi_2 := [c](\varphi_1 \rightarrow \varphi_2) \wedge \neg[c]\Delta_{\sigma(\varphi_2)}(\varphi_1 \rightarrow \varphi_2) \quad (1.17)$$

$$\varphi_1 \Rightarrow_{c,\Phi}^{co} \varphi_2 := [c] \bigwedge \Phi \wedge [-c] \neg \bigwedge \Phi \quad \text{with } \varphi_1 \rightarrow \varphi_2 \in \Phi \quad (1.18)$$

Formula 1.16 defines counts-as as a contextual implication which is not valid in the model $\neg[U](\varphi_1 \rightarrow \varphi_2)$. Formula 1.17, along a similar line, defines counts-as as a contextual implication which would not be valid in the context any more if we ‘release’ or ‘forget’ the vocabulary of the consequent. Finally, Formula 1.18 defines counts-as as a contextual implication belonging to a set Φ which defines the context of reference c . The definition of the context is rendered by stating that all formulae in Φ are valid in c — $[c] \bigwedge \Phi$ —and that some are false in the complement of c — $[-c] \neg \bigwedge \Phi$.

¹³“A set of constitutive rules defines a logical space” [Ricciardi, 1997, p.6].

The work of Grossi *et al.* has provided a detailed study of the structural properties of all the introduced operators¹⁴ and, also, of their relative logical strength. In particular, the following logical relations hold between the different forms of counts-as:

$$\Rightarrow_{c,\Phi}^{co} \subset \Rightarrow_c^{As} \subset \Rightarrow_c^{cl+} \subset \Rightarrow_c^{cl} . \quad (1.19)$$

Furthermore, all the logics used to define the above operators have been proven sound and strongly complete.

So, the approach of Grossi *et al.* has focused on the identification of different meanings of the counts-as locution, and has formalized those meanings within the framework of normal modal logic. This perspective determined significant differences with respect to the formalization proposed by Jones *et al.* and it is worth illustrating these differences by showing how the formalization of Grossi *et al.* handles Example 1.2 in contrast to the formalization provided in Example 1.3.

EXAMPLE 1.4 (Institutional detachment in Grossi *et al.*) *Let us now denote with Φ the rules of the institution at issue. This set Φ contains the implication $p \rightarrow m$, and Φ defines a context c . We have then a constitutive counts-as: $p \Rightarrow_{c,\Phi}^{co} m$. Now, suppose we are in a situation, let us call it s , in which the officer pronounces the couple husband and wife (p). We have two possibilities. If s belongs to the context defined by Φ we can conclude that in s the couple is married (m) by the classificatory counts-as: $p \Rightarrow_c^{cl} m$ which follows from the constitutive one (recall Formula 1.19). In other words, if we are indeed in a situation where context c applies, then we can conclude that the couple is married. Note that this is an alternative version of the institutional detachment of Example 1.3, albeit semantical.¹⁵*

$$\{p \Rightarrow_{c,\Phi}^{co} m, \bigwedge \Phi, p\} \models m \quad (1.20)$$

On the other hand, if the context does not apply, i.e., if we drop assumption $\bigwedge \Phi$, the conclusion can no longer be drawn.

Examples 1.3 and 1.4 nicely illustrate one of the most striking differences between the approaches of Jones *et al.* and Grossi *et al.*, which consists precisely in the rendering of institutional detachment: Formula 1.9 vs. Formula 1.20. As Examples 1.3 and 1.4 nicely show, the key difference between the two approaches consists in the consequences that can be drawn from the

¹⁴We will come back to this aspect in Section 5.

¹⁵A fully syntactical version can be provided by introducing nominals (see [Grossi, 2007; Grossi *et al.*, 2008]).

assumption of the ‘factual’ truth p . In Jones *et al.* the truth of p always determines, via counts-as, the institutional truth of m or, more precisely, the truth of $D_c m$ (Formula 1.9). In Grossi *et al.*, instead, p does not determine the truth of a modalized occurrence of m , but of m itself, although this entailment is conditional under the assumption that the rules defining the context of the counts-as are actually in force (Formula 1.20). This is rather interesting and reveals a radical difference in the basic set up of the formalization of counts-as conditionals ultimately concerning the rendering of a notion of institutional truth. In Jones *et al.* institutional truth is represented by the modality D_c . So, m is institutionally true in a state s if $\mathcal{M}, s \models D_c m$. On the other hand, in Grossi *et al.* institutional truth can be viewed just as standard truth (i.e., satisfaction in a pointed model) where the evaluation state belongs to the context defined by the set of formulae Φ , that is, $\mathcal{M}, s \models \bigwedge \Phi \wedge m$ and m belongs to the vocabulary of Φ .

3.4 Lorini *et al.*

Lorini *et al.* follow a research line similar to the one pursued by Grossi *et al.*, but focus on an aspect of constitutive rules which was neglected in the latter analysis, namely the fact that constitutive rules, in order to be in force, need to be *accepted* by the members of the society concerned. To pursue this aim, they propose a study of counts-as conditionals where the basic building block is not a normal logic of context, as in Grossi *et al.*, but a logic of acceptance by groups of agents in a set N who are members of institutions in a set C . The primitive modal operator in this logic is $\mathcal{A}_{X:c}$, whose intuitive reading is: “all agents in group X acting as members of institution C accept that ...”. The operator is interpreted according to the standard satisfaction relations in Kripke semantics:

$$\mathcal{M}, s \models \mathcal{A}_{X:c} \varphi \iff \forall s' \text{ s.t. } s' \in ACC_{X:c}(s) : \mathcal{M}, s' \models \varphi \quad (1.21)$$

where $c \in C$, $\emptyset \subset X \subseteq N$, $ACC_{X:c}$ is a function assigning to each state the set of states ‘accepted’ by X as members of c . We will not provide the formal properties of that function, but for our purposes it suffices to say that a set $ACC_{X:c}(s)$ behaves in a slightly weaker way than a context in Grossi *et al.*, as it represents some kind of collective mental attitude of a group of agents, rather than an external objective institutional reality.

This said, Lorini *et al.* refine the pattern of Grossi *et al.* proper classificatory rules to define a counts-as conditional based on acceptance:

$$\varphi_1 \triangleright_c \varphi_2 \quad := \quad \left(\bigwedge_{\emptyset \subset X \subseteq N} \mathcal{A}_{X:c}(\varphi_1 \rightarrow \varphi_2) \right)$$

$$\wedge \neg \left(\bigwedge_{c' \in C} \left(\bigwedge_{\emptyset \subset Y \subseteq N} \mathcal{A}_{Y:c'}(\varphi_1 \rightarrow \varphi_2) \right) \right) \quad (1.22)$$

In other words, φ_1 counts as φ_2 in c if and only if all groups of agents X acting as members of c accept that φ_1 implies φ_2 , and there exists at least one institution c' and group of agents Y which does not accept the implication.

Lorini *et al.* provide a thorough analysis of the definition in Formula 1.22 as well as soundness and completeness results for the logic of acceptance.

4 Alternative formalisms for counts-as

To this group belong the works by Governatori *et al.*, based on defeasible logic, by Boella *et al.*, based on Input/Output logic and by Lindahl *et al.*, which makes use of an algebraic formalism.

4.1 Governatori *et al.*

Governatori *et al.* tackle the analysis of constitutive rules and counts-as conditionals from a legally-informed point of view. In particular, they stress the importance of incorporating in the analysis the typical feature of legal reasoning known as defeasibility which, in the case of counts-as, roughly amounts to the following observation: the inference from X to Y via a statement “ X counts as Y in context C ” need not be logically valid, and it can be retracted in the presence of further information (e.g., if X is of a somehow exceptional kind). In short, they propose an analysis of counts-as allowing for defeasible institutional detachment (see Example 1.5 below).

This aim is achieved within the framework of defeasible logic [Nute, 1987] whose key structure is the so-called defeasible theory, that is, a triple $(F, R, >)$ where:

- F is a finite set of literals representing basic facts;
- R is a set of rules including *strict rules*, whose conclusions are indisputable, *defeasible rules*—denoted \Rightarrow —whose conclusions can be defeated, and *defeaters* which are essentially statements to the effect that a given defeasible rule is not applicable;
- $>$ is a priority relation among rules whose function is to resolve conflicts between rules.

The core intuition of Governatori *et al.* consists in representing institutional scenarios such as Example 1.2 as defeasible theories where F represents the set of facts and where counts-as conditionals are rendered as defeasible

rules.¹⁶ To illustrate this idea, we resort to our running example, Example 1.2, by adapting examples to be found in [Governatori and Rotolo, 2008].

EXAMPLE 1.5 (Defeasible institutional detachment) *Consider the scenario given in Example 1.2, and assume also that “the officer performing the wedding is under threat of death by the couple” (in symbols, d). We have the following defeasible theory $(F, R, >)$ with:*

- $F = \{p, d\}$
- $R = \{r_1 : p \Rightarrow m; r_2 : p, d \Rightarrow \neg m\}$
- $> = \{(r_2, r_1)\}$

In the terminology of defeasible logic, from this defeasible theory we can defeasibly prove that $\neg m$, while conclusion m obtainable via rule r_1 is overridden by rule r_2 .

A further important characteristic of this approach, with respect to those presented above, consists in its tractability from a computational point of view.

4.2 Boella *et al.*

Like the work by Governatori *et al.* the analysis of counts-as proposed in the series of papers [Boella and van der Torre, 2003; Boella and Van der Torre, 2004; Boella and van der Torre, 2006] is also an attempt to formalize constitutive rules by means of techniques coming from the area of defeasible reasoning. The technique is, in this case, the so-called input/output logic (IOL) [Makinson and van der Torre, 2000].

The key idea behind the application of IOL to the analysis of counts-as, consists in representing constitutive norms simply as ordered pairs (a, b) where a represents the antecedent of the rule, and b its consequent: “ a counts as b ”. Typically, both a and b are taken to be formulae from propositional logic. Each set of such ordered pairs can be seen as an inferential mechanism which, given an input, determines an output based on such rules.

Various definitions can be given of how to produce the output on the basis of a set of pairs, and all consist in ways of closing the given set of pairs by adding new pairs in accordance to some principles, of which we give two examples:

$$SI : \frac{(a, b)}{(a \wedge c, b)} \quad CT : \frac{(a, b), (a \wedge b, c)}{(a, c)} \quad (1.23)$$

¹⁶Governatori *et al.* propose actually an articulated extension of defeasible theories, but here we want to expose just the basic idea underlying their approach leaving the details to the interested reader.

where *SI* stands for strengthening of the input—essentially an antecedent strengthening property—and *CT* stands for cumulative transitivity. Formally, given a set *CONS* of pairs, a closure operation *C* defined in terms of some of the above principles, and a set of facts *A*, the output of *CONS* given *C* and a set of input formulae *I* is:

$$out_C(CONS, A) = \{b \mid (a, b) \in C(CONS) \text{ and } s \in A\} \quad (1.24)$$

The freedom available in defining the output operation makes IOL an extremely versatile framework. As to the analysis of counts-as, Boella *et al.* usually employ the output operation which uses a closure based only on the two above principles *SI* and *CT*. The technical name in the IOL literature for such an output operation is *simple-minded reusable output*.

The work of Boella *et al.* does not focus further on the study of structural properties of counts-as statements as such, but is rather interested in the application of such statements to the formal specification of multi-agent systems. In particular, the authors focus on the interaction between the IOL representation of constitutive norms and the representation of regulative norms in the same logic.¹⁷ The representation of regulative norms follows the very same formulation: regulative norms are pairs (d, e) of conditions and normative consequences, and a set of such norms, under a given closure operation, can be used to yield the set of normative consequences of a given set of propositional formulae.

This simple approach naturally lends itself to a formal representation of the sort of nesting of constitutive norms (from brute to institutional facts) and regulative norms (from institutional facts to normative consequences). So, given a set of pairs *CONS* representing constitutive norms, and a set of pairs *REG* representing regulative norms, the set of normative consequences of a given set of facts *A* is determined by the nested application of an output operation:

$$out_{C'}(REG, out_{C''}(CONS, A)) \quad (1.25)$$

where *C'* is the set of principles for the output operation on regulative norms, and *C''* the set of principles for the output operation on constitutive norms (like in Formula 1.23). Boella *et al.* develop this intuition further to more complex forms of interaction between the two output operations, but Formula 1.25 provides the basic idea.

Finally, it is worth mentioning that the authors assume an original conceptualization of normative systems as the set of beliefs and desires of the

¹⁷The idea that regulative and constitutive rules should be formalized in terms of the same logic is also argued for in [Gelati *et al.*, 2002].

system itself viewed as one agent, where constitutive rules represent the system’s beliefs, and the normative rules its desires.

4.3 Lindahl *et al.*

Based on work by the authors from the late 90s [Lindahl and Odelstad, 2000] in which they advocated an algebraic analysis of normative systems, the series of papers [Lindahl and Odelstad, 2006; Lindahl and Odelstad, 2008a; Lindahl and Odelstad, 2008b] focuses on ‘intermediate concepts’, that is, the kind of concepts such as “ownership”, or “marriage”, that are most typically introduced via counts-as statements. In focusing on this issue, the authors provide a formal account of counts-as that highlights its character of ‘technique of presentation’ which has been discussed earlier in this chapter in Section 2.3.

The approach chosen by Lindahl *et al.* is very close in spirit to the one, discussed above, of IOL. However, the formal machinery deployed is considerably more complex as it hinges on several algebraic and order-theoretical notions. In this section we provide just a brief sketch of the basic technical ideas underlying the framework, referring the interested reader to the authors’ chapter in this same volume for more details. Furthermore, as the framework has undergone several modifications, it is appropriate to mention that our presentation will be based, specifically, on the recent paper [Lindahl and Odelstad, 2008a].

The key notion in Lindahl *et al.*’s work is the one of *Boolean joining system*. The idea behind it is that norms can be seen—exactly as in IOL—as simple pairs (a, b) connecting (factual) conditions to (normative) consequences. Both conditions and consequences are viewed as structured according to a supplemented Boolean algebra, that is, a structure $B = \langle X, \sqcap, -, \perp, \rho \rangle$ where $\langle X, \sqcap, -, \perp \rangle$ is a Boolean algebra, and $\rho \subseteq X^2$ a binary relation including the order \preceq associated to the Boolean algebra.¹⁸ Relation ρ captures, intuitively, a relation of entailment between the elements of the algebra, which strengthens the classical entailment relation given by \preceq . Now, once conditions and consequences are represented via such structures, a *Boolean joining system* is a structure $\langle B_1, B_2, J \rangle$ where B_1 and B_2 are supplemented Boolean algebras for conditions and, respectively, consequences, and J is a set of pairs (b_1, b_2) *joining* elements of B_1 with elements of B_2 . That set, which is required to satisfy further conditions which we refrain from mentioning here, represents the stipulation of

¹⁸A partial linear order can always be associated to a given Boolean algebra as follows:

$$a \preceq b \iff a \sqcap b = a. \quad (1.26)$$

the (regulative) norms of a given normative system.

So how are constitutive norms represented in this framework? The idea is that such norms introduce *intervenients* of existing joinings (b_1, b_2) , where an intervenient is an element b of a supplemented subalgebra B of B_1 and B_2 such that b_1 entails b —in symbols, $b_1 \rho b$ —and b entails b_2 —in symbols, $b \rho b_2$ —and in addition b_1 is, roughly, the weakest ground for b and b_2 the strongest consequence of it. According to Lindahl *et al.* the notion of interpolant characterizes the structural properties of constituted concepts such as “ownership” within a given normative system.¹⁹

5 Classifying the formal approaches to counts-as

The previous sections have provided a somewhat historical overview of the development of the theory of counts-as and constitutive rules. The present section proposes a systematization of these different contributions according to a few different criteria. The various approaches are compared with one another according to the aspects of constitutive rules they deal with, the method they follow in pursuing their analysis, and the formal properties of the resulting models.

Before starting this section, we want to stress that our aim here is not to argue in favour of or against any of the approaches presented, but rather to provide some useful guidelines for the reader to navigate the existing literature.

5.1 Thematic classification

Each of the formal approaches to counts-as we have presented focus their analysis on some features of constitutive rules, abstracting from others. In particular, for each of them it is easy to recognize one main focus of attention in the development of the analysis.

- Contextual aspects of counts-as. These have been stressed since the Jones *et al.* work and the reference to a context has been recognized by almost all approaches as essential for the syntax of counts-as conditionals.
- Classificatory aspects of counts-as. These have been highlighted in particular in [Grossi *et al.*, 2006a; Grossi *et al.*, 2008].
- Counts-as and actions (counts-as as the basis of institutional power). The grounding of institutional power on counts-as statements was one of the key issues stressed in [Jones and Sergot, 1996].

¹⁹Lindahl *et al.*'s approach is treated in detail in a dedicated chapter in this handbook.

- Counts-as and conventions. The relation between counts-as and convention, in particular within communication theory, has received attention in [Jones and Parent, 2004; Jones and Parent, 2007].
- Counts-as as grounded on dedicated agents’ mental attitudes. This topic has been systematically investigated, within modal logic, in [Gaudou *et al.*, 2008; Lorini and Longin, 2008; Lorini *et al.*, 2009].
- Counts-as as related to regulative norms. The topic of how regulative norms (e.g., obligations, permissions, etc.) are related to constitutive ones—a topic much discussed in Searle’s work [Searle, 1995]—has been studied, in particular, in [Boella and Van der Torre, 2004; Boella and van der Torre, 2006].
- Counts-as as related to the definition of legal terms (e.g., contract) in legal systems. This aspect is highlighted in [Lindahl and Odelstad, 2006; Lindahl and Odelstad, 2008a; Lindahl and Odelstad, 2008b].

Table 1.1 on page 24 compactly records the thematic focuses of each approach to counts-as considered in this overview.

5.2 Methodological classification

The formal analysis of counts-as and constitutive rules is an exercise in applied logic or, more broadly, in applied mathematics.²⁰ We recognize three salient methodological features of the formal approaches to counts-as discussed in this chapter. Such features are not properties of the formal analysis of counts-as alone, but are common to any logico-mathematical analysis of informal philosophical notions.

Object vs. meta-language

The analysis of counts-as is either carried out within the object language, by means of dedicated operators, or within the meta-language, by characterizing dedicated notions of non-classical logical consequence.

²⁰It might be instructive to recall an excerpt from [Tarski, 1944] neatly describing how logic is applied to the analysis of concepts:

“[...] it seems to me obvious that the only rational approach to such problems [of concept analysis] would be the following: [1] We should reconcile ourselves with the fact that we are confronted, not with one concept, but with several different concepts which are denoted by one word; [2] we should try to make these concepts as clear as possible (by means of definition, or of an axiomatic procedure, or in some other way); [3] to avoid further confusions, we should agree to use different terms for different concepts; and then we may proceed to a quiet and systematic study of all concepts involved, which will exhibit their main properties and mutual relations” ([Tarski, 1944], p. 355).

	Grossi <i>et al.</i>	Lorini <i>et al.</i>	Jones <i>et al.</i>	Gelati <i>et al.</i>
Contexts	✓	✓	✓	✓
Classification	✓	✓		
Power			✓	✓
Conventions			✓	
Mental attitudes				
Const. vs. reg.	✓	✓	✓	
Legal concepts	✓	✓		

	Governatori <i>et al.</i>	Lindhahl <i>et al.</i>	Boella <i>et al.</i>
Contexts			
Classification			
Power			
Conventions			
Mental attitudes			✓
Const. vs. reg.	✓	✓	✓
Legal concepts	✓	✓	

Table 1.1. An inventory of the themes addressed in the literature on counts-as conditionals

At the object level, the analysis moves from a given logic whose language is expanded with a suitable operator—in the case of counts-as conditionals, typically, a ternary operator with places for antecedent, consequent and context—which denotes the to-be-analyzed notion. The new operator is then studied, axiomatically or semantically, within the framework given by the background logic—e.g. normal modal logic [Blackburn *et al.*, 2001] or classical modal logic [Chellas, 1980]. In the meta-language case, instead, a background logic for the analysis is selected—in the case of counts-as, typically, propositional logic—but its language is not expanded. Instead, the notion of logical consequence or, equivalently, of derivation of the original logic is strengthened or weakened in order to capture certain features which are considered characteristic of the reasoning involved with the to-be-analyzed notion. Roughly, while approaches of the first type are interested in the logic of statements of the sort “ φ counts as ψ in context c ”, approaches of the second type are interested in which ψ s can be inferred by assuming φ and a given set c of constitutive rules.

Defined vs. primitive

Be it studied as an operator or as a relation of logical consequence (or derivability), counts-as is formally characterized either by defining it in terms of simpler components (that is, simpler logical operators, respectively, simpler logical relations) whose logic is already available, or assuming it as a logical primitive and studying it in its own right. Approaches of the first type are reductionistic in the sense that they consider statements “ φ counts as ψ in context c ” to be synonymous with other statements (e.g., “ φ implies ψ in context c ” [Grossi *et al.*, 2006a]), thereby reducing the logic of counts-as to the interaction of logics of simpler components (in this case the logics of “implies” and “in context”).

Syntactic (axiomatic) vs. semantic (model-theoretic)

All the formal frameworks considered in the chapter, with few exceptions, provide both a proof-theory and a semantics for the logic of counts-as. However, differences arise regarding which one of the two perspectives is privileged during the initial set up of the formal theory. If a semantics is fixed, then a sound and complete axiomatization is looked for and, vice versa, if an axiomatics is established, then a semantics is looked for with respect to which the axiomatics is sound and complete.

In the first case the formal analysis moves initially from insights concerning the set up of the formal models on which counts-as statements can be interpreted. For example, as in the case of Grossi *et al.*, the assumption that counts-as statements are essentially of a classificatory type, leads the authors to use models on which counts-as statements are interpreted, essen-

	Level of analysis	Characterization	Intuition
Boella <i>et al.</i>	metalanguage	primitive	syntactic
Jones <i>et al.</i>	object language	primitive	syntactic
Gelati <i>et al.</i>	object language	primitive	syntactic
Governatori <i>et al.</i>	metalanguage	primitive	syntactic
Grossi <i>et al.</i>	object language	defined	semantic
Lindahl <i>et al.</i>	metalanguage	primitive	syntactic
Lorini <i>et al.</i>	object language	defined	semantic

Table 1.2. Methodological classification of formal approaches to counts-as.

tially, as concept subsumptions. So, these approaches develop their formal analysis by first establishing under what conditions the to-be-formalized statements are true in the dedicated models (e.g., if and only if “all φ -states in context c are ψ -states” [Grossi *et al.*, 2006a]). The proof-theoretic properties of the various statements are studied as a consequence of the semantic set up.

In the second case, the analysis is driven instead by considering plausible-looking candidates for logical truths as axioms, by looking at the natural language counterparts of the to-be-formalized statements. Driving questions are, in this case, whether, for instance, counts-as statements are reflexive, or transitive, or symmetric, etc. Once the set of axioms is fixed, then suitable models are developed, on which the axiomatized statements can be interpreted.

The distinction between syntactic (axiomatic) and semantic (model-theoretic) is a very typical methodological dichotomy to be found in work in applied logic, and the analysis of counts-as is no exception.²¹

Table 1.2 shows how the various approaches discussed in this chapter can be placed with respect to the methodological standpoints described above.

²¹We will not discuss in this chapter what the pros and cons are of each of these approaches. Interesting considerations on this issue can be found in [Tarski, 1944; Tarski, 1983].

5.3 Logical classification

The present section compares the various approaches introduced in Section 3 from the point of view of the formal frameworks specifically used in the analysis.

First we look at the structural properties of counts-as conditionals in those approaches that deal with them at an object-language level (see Table 1.2). From a technical point of view, this is arguably the most informative comparison and it expands the comparison that was first elaborated in [Grossi, 2007; Grossi *et al.*, 2008]. As Section 3 has made clear, the formal analysis of counts-as has been pursued in rather different—and thus difficult to compare—logical paradigms. The two technically closest approaches are the ones of Grossi *et al.* and Lorini *et al.*, which work in normal modal logic. This section compares the relative strength of the logics presented in those works, and also relates them to the non-normal modal logic of, respectively, Gelati *et al.*, and Jones *et al.*

Structural properties of counts-as conditionals

Conditional \Rightarrow_c^{cl} enjoys strong properties (in particular reflexivity, antecedent strengthening, and transitivity) and displays, therefore, a very classical behavior. Instead, conditional \Rightarrow_c^{cl+} behaves much less classically, rejecting reflexivity, strengthening of the antecedent, even the weaker version of cautious monotonicity, and transitivity. On the other hand, it still retains a weaker form of transitivity, namely cumulative transitivity.

As we have seen in Section 3, Jones *et al.* have developed a logic for counts-as conditionals (denoted by the operator \Rightarrow_c) obeying the following principles: left logical equivalence, right logical equivalence, disjunction of antecedents, conjunction of the consequents and transitivity. Recall, though, that it does not enjoy cumulative transitivity and cautious monotonicity.

Gelati *et al.* argue, instead, that the logic of counts-as conditionals, which they denote via the operator \Rightarrow , amounts to the logic of preferential reasoning [Kraus *et al.*, 1990], preferential reasoning being characterized by the following properties: reflexivity, left logical equivalence, weakening of the consequent, conjunction of the consequents, cut, cautious monotonicity and disjunction of the antecedents.

An overview of the main properties enjoyed by each object-level formal characterization of counts-as is provided in Table 1.3.

This overview provides grounds for a number of interesting observations. First of all, notice that there seems to be a structural hard core of all characterizations of counts-as including Grossi *et al.*, which corresponds to properties (D) to (G) inclusive in Table 1.3. These properties are exactly

the ones recognized as a sort of minimal characterization of counts-as in [Jones and Sergot, 1996]. There are then two remarkable facts to be noticed, which concern the relation between our notions of contextual and proper contextual classification and the notions of counts-as axiomatically characterized by Gelati *et al.* and Jones *et al.*

First, the notion of counts-as statements as conditional counterparts of preferential reasoning (\Rightarrow) represents a defeasible form of contextual classification (\Rightarrow_c^{cl}), since the only properties distinguishing the two notions are strengthening of the antecedent (B) and transitivity (C), which in the presence of reflexivity (A) and cut (I) are actually equivalent (see [Kraus *et al.*, 1990]). From a semantic point of view, this constitutes a very interesting fact. In a way, it allows us to attach a precise meaning to the notion of counts-as axiomatized by Gelati *et al.* : if the statement “X counts-as Y in context C”, intended as contextual classification, means “X is classified as Y in C”, then the same statement read in the fashion of Gelati *et al.* would mean “X is classified as Y in C, *modulo exceptions*”, or “it *normally* follows from C that X is classified as Y”.

Second, the notions of proper contextual classification (\Rightarrow_c^{cl+}) and of acceptance-based counts-as (\triangleright_c) both appear to correspond to a slightly weaker version of the counts-as conditional proposed by Jones *et al.* (\Rightarrow_c) where transitivity (C) is substituted by the weaker property of cumulative transitivity (I).

Relative strength of modal counts-as

A final comparison is worth making, which focuses on those approaches to counts-as that are based on standard modal logic. Among all the logics of counts-as, the ones studied by Grossi *et al.* (see Section 3.3) and Lorini *et al.* (see Section 3.4) are the ones bearing the most similarities. In fact, [Lorini *et al.*, 2009] has proven an embedding of the logic of proper classificatory counts-as in a version of AL strengthened with suitable axioms.

Figure 1.4 displays the relative logical strength of the counts-as conditionals investigated by Grossi *et al.* and Lorini *et al.*, and relates them to the ones proposed by Jones *et al.* and by Gelati *et al.*. Proper contextual classification \Rightarrow_c^{cl+} can be viewed as an extension of both contextual classification (\Rightarrow_c^{cl}) and acceptance-based counts-as (\triangleright_c). In turn, it is strengthened independently by ascriptive counts-as (\Rightarrow_c^{as}) and constitutive counts-as (\Rightarrow_c^{co}).

6 Some open problems

We conclude the chapter by pointing to a few open research questions in the formal analysis of counts-as and constitutive norms, which we consider most interesting and urgent.

		\Rightarrow_c^{cl}	\Rightarrow_c^{cl+}	\Rightarrow_c	\Rightarrow	\triangleright_c
A	Reflexivity	✓			✓	
B	Antecedent Strengthening	✓				
C	Transitivity	✓		✓		
D	Disjunction of the Antecedents	✓	✓	✓	✓	✓
E	Conjunction of the Consequents	✓	✓	✓	✓	✓
F	Left Logical Equivalence	✓	✓	✓	✓	✓
G	Right Logical Equivalence	✓	✓	✓	✓	✓
H	Consequent Weakening	✓			✓	
I	Cumulative Transitivity	✓	✓		✓	✓
L	Cautious Monotonicity	✓			✓	

Table 1.3. Properties of counts-as operators, where: \Rightarrow_c^{cl} is the contextual classification of Grossi *et al.* ; \Rightarrow_c^{cl+} the proper contextual classification of Grossi *et al.* ; \Rightarrow_c is the counts-as conditional of Jones *et al.* ; \Rightarrow is the counts-as conditional of Gelati *et al.* . ; and \triangleright_c is the acceptance-based conditional of Lorini *et al.* .

First, the relation of logics of counts-as with logics of action. The chapter has not touched upon this issue which, although extensively highlighted by Jones *et al.*, has not been thus far systematically addressed by the literature in the field. As a matter of fact, in Jones *et al.* the thrust towards a formal theory of counts-as comes from an attempt to provide a comprehensive formal theory of institutional action, accounting for key phenomena of institutional reality such as institutional power.

Second, a key aspect of constitutive norms which still awaits a thorough formal analysis is the aspect which might be called ‘language creation’. This aspect has only partially been addressed in [Grossi, 2010] (see Section 3.3) and concerns the capacity that constitutive norms have of, literally, creating new words. Constitutive norms not only define terms but, in a way, they really expand the current language of the normative system at issue. From a logical point of view, this poses several interesting technical questions whose answers would bear definite relevance for a full understanding of constitutive

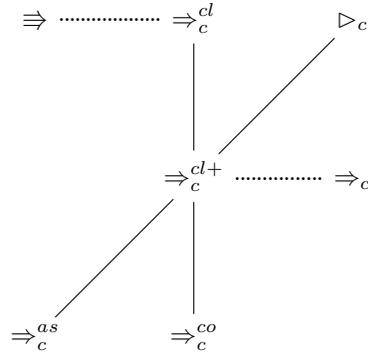


Figure 1.4. Diagram of all modal counts-as operators. Lines link weaker conditionals (top) to stronger ones (bottom). Dotted lines link the counts-as conditionals defined in the logics of Grossi *et al.* and Lorini *et al.* with conditionals defined in non-standard modal logics (in this case by Gelati *et al.*, \Rightarrow_c and Jones *et al.*, \Rightarrow_c) to which they bear structural similarities (recall Table 1.3).

norms.

Third, the comparison of the properties of counts-as conditionals and causal conditionals. It has been claimed in, e.g., [Searle, 1969] and [Jones and Parent, 2007] that counts-as conditionals are central to understanding the foundations of interpersonal communication. However, as we see for instance in [Grice, 1957], a contrast is often drawn between signalling mechanisms that exploit causal connections (Grice’s so-called natural meaning), and those that are characteristic of human communication. The latter were those for which Grice developed his intention-based account of non-natural meaning, but arguably the contrast is better understood in terms of the distinction between non-conventional and conventional meaning—see, e.g., [Lewis, 1969] and the recent work on signalling—and the natural processes that create conventions—by Brian Skyrms [Skyrms, 2010]. It may be that some further light can be thrown on this distinction by a comparison of the respective properties of counts-as and causal conditionals.

Finally, on the wave of attention to normative change that is characterizing much of the recent research in deontic logic²², some works have been focusing on the dynamics of constitutive norms. In particular: [Aucher

²²See dedicated chapter in this handbook.

et al., 2009] has provided an analysis, based on a dynamic variant of the classificatory logic of counts-as [Grossi *et al.*, 2006a], of how counts-as conditionals can be introduced or removed from the specification of a context, thereby offering a very stylized formal model of the enactment and abrogation of constitutive norms; [Boella *et al.*, 2010] has interestingly argued for the formal study of the dynamics of constitutive norms as a privileged means to provide a solid understanding of the key judicial phenomenon of legal interpretation. Although these works provide some important first steps in the formal understanding of the dynamics of constitutive rules, much still remains unexplored.

7 Conclusions

The chapter has provided a detailed overview of that branch of deontic logic which, in the last fifteen years, has addressed the issue of the formal analysis of constitutive norms and counts-as conditionals.

The chapter has addressed some of the philosophical/conceptual features of the various approaches available in the literature and their key technical aspects. The resulting overview has then been used—in the key section of the chapter (Section 5)—to provide a systematic comparison of the various approaches along three different lines: the different aspects addressed in the formal analysis, the methodology chosen for delivering the analysis, and the formal relationships between the different logical systems.

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BIBLIOGRAPHY

- [Alchourrón and Bulygin, 1971] C. E. Alchourrón and E. Bulygin. *Normative Systems*. Springer Verlag, 1971.
- [Anscombe, 1958] G. E. M. Anscombe. On brute facts. *Analysis*, 18:69–72, 1958.

- [Artosi *et al.*, 2004] A. Artosi, A. Rotolo, and S. Vida. On the logical nature of counts-as conditionals. In C. Cevenini, editor, *The Law and Electronic Agents: Proceedings of the LEA 04 workshop*, pages 9–34, Bologna, Italy, 2004.
- [Aucher *et al.*, 2009] G. Aucher, D. Grossi, A. Herzig, and E. Lorini. Dynamic context logic. In X. He, J. Horty, and E. Pacuit, editors, *Proceedings of LORI 2009*, volume 5834 of *LNAI*. Springer, 2009.
- [Blackburn *et al.*, 2001] P. Blackburn, M. de Rijke, and Y. Venema. *Modal Logic*. Cambridge University Press, Cambridge, 2001.
- [Boella and van der Torre, 2003] G. Boella and L. van der Torre. Attributing mental attitudes to normative systems. In *AAMAS '03: Proceedings of the second international joint conference on Autonomous agents and multiagent systems*, pages 942–943, New York, NY, USA, 2003. ACM Press.
- [Boella and Van der Torre, 2004] G. Boella and L. Van der Torre. Regulative and constitutive norms in normative multiagent systems. In D. Dubois, C. A. Christopher A. Welty, and M. Williams, editors, *Proceedings of KR2004, Whistler, Canada*, pages 255–266, 2004.
- [Boella and van der Torre, 2006] G. Boella and L. van der Torre. A logical architecture of a normative system. In L. Goble and J.-J.Ch. Meyer, editors, *Deontic Logic and Artificial Normative Systems: 8th International Workshop on Deontic Logic in Computer Science (DEON 2006)*, volume 4048 of *LNCS*, pages 24–35, 2006.
- [Boella *et al.*, 2010] G. Boella, G. Governatori, A. Rotolo, and L. van der Torre. A logical understanding of legal interpretation. In L. Fangzhen, U. Sattler, and M. Truszczynski, editors, *Proceedings of the 12th International Conference on the Principles of Knowledge Representation and Reasoning (KR 2010)*, pages 563–565, 2010.
- [Bulygin, 1992] E. Bulygin. On norms of competence. *Law and Philosophy 11*, pages 201–216, 1992.
- [Chellas, 1980] B. F. Chellas. *Modal Logic. An Introduction*. Cambridge University Press, Cambridge, 1980.
- [Cherry, 1973] C. Cherry. Regulative rules and constitutive rules. *The Philosophical Quarterly*, 23(93):301–315, 1973.
- [Fornara *et al.*, 2007] N. Fornara, F. Viganò, and M. Colombetti. Agent communication and artificial institutions. *Journal of Autonomous Agents and Multi-Agent Systems*, 14:121–142, 2007.
- [Gaudou *et al.*, 2008] B. Gaudou, D. Longin, E. Lorini, and L. Tummolini. Anchoring institutions in agents’ attitudes: Towards a logical framework for autonomous multiagent systems. In L. Padgham, D. Parkes, J. Müller, and S. Parsons, editors, *Proceedings of the 7th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2008)*, pages 728–735. ACM, 2008.
- [Gelati *et al.*, 2002] J. Gelati, G. Governatori, A. Rotolo, and G. Sartor. Actions, institutions, powers. Preliminary notes. In *International Workshop on Regulated Agent-Based Social Systems: Theories and Applications (RASTA'02)*, pages 131–147, 2002.
- [Gelati *et al.*, 2004] J. Gelati, A. Rotolo, G. Sartor, and G. Governatori. Normative autonomy and normative co-ordination: Declarative power, representation, and mandate. *Artif. Intell. Law*, 12(1-2):53–81, 2004.
- [Goldman, 1976] A. I. Goldman. *A Theory of Human Action*. Princeton University Press, Princeton, 1976.
- [Governatori and Rotolo, 2008] G. Governatori and A. Rotolo. A computational framework for institutional agency. *Artificial Intelligenc and law*, 16:25–52, 2008.
- [Grice, 1957] H. P. Grice. Meaning. *Philosophical Review*, 66:377–388, 1957.
- [Grossi *et al.*, 2005] D. Grossi, J.-J.Ch. Meyer, and F. Dignum. Modal logic investigations in the semantics of counts-as. In *Proceedings of the Tenth International Conference on Artificial Intelligence and Law (ICAIL'05)*, pages 1–9. ACM, June 2005.
- [Grossi *et al.*, 2006a] D. Grossi, J.-J.Ch. Meyer, and F. Dignum. Classificatory aspects of counts-as: An analysis in modal logic. *Journal of Logic and Computation*, 16(5):613–643, 2006. Oxford University Press.

- [Grossi *et al.*, 2006b] D. Grossi, J.-J.Ch. Meyer, and F. Dignum. Counts-as: Classification or constitution? An answer using modal logic. In L. Goble and J.-J.Ch. Meyer, editors, *Proceedings of DEON 2006*, LNAI 4048, pages 115–130, 2006.
- [Grossi *et al.*, 2008] D. Grossi, J.-J.Ch. Meyer, and F. Dignum. The many faces of counts-as: A formal analysis of constitutive-rules. *Journal of Applied Logic*, 6(2):192–217, 2008.
- [Grossi, 2007] D. Grossi. *Designing Invisible Handcuffs. Formal Investigations in Institutions and Organizations for Multi-agent Systems*. PhD thesis, Utrecht University, SIKS, 2007.
- [Grossi, 2008] D. Grossi. Pushing Anderson’s envelope: The modal logic of ascription. In R. van der Meyden and L. van der Torre, editors, *Proceedings of the 9th International Conference on Deontic Logic in Computer Science (DEON 2008)*, number 5076/2008 in LNAI, pages 263–277. Springer, 2008.
- [Grossi, 2009] D. Grossi. A note on brute vs. institutional facts: Modal logic of equivalence up to a signature. In Guido Boella, Pablo Noriega, Gabriella Pigozzi, and Harko Verhagen, editors, *Normative Multi-Agent Systems*, volume 09121 of *Dagstuhl Seminar Proceedings*, Dagstuhl, Germany, 2009.
- [Grossi, 2010] D. Grossi. Norms as ascriptions of violations: An analysis in modal logic. *Journal of Applied Logic (to appear)*, 2010.
- [Hindriks, 2009] F. Hindriks. Constitutive rules, language, and ontology. *Erkenntnis*, 71:235–275, 2009.
- [Jones and Parent, 2004] A. J. I. Jones and X. Parent. Convention signaling acts and conversation. In F. Dignum, editor, *Proceedings of ACL 2003*, pages 1–17. Springer, 2004.
- [Jones and Parent, 2007] A. J. I. Jones and X. Parent. A convention-based approach to agent communication languages. *Group Decision and Negotiation*, 16:101–141, 2007.
- [Jones and Sergot, 1996] A. J. I. Jones and M. Sergot. A formal characterization of institutionalised power. *Journal of the IGPL*, 3:427–443, 1996.
- [Krabbendam and Meyer, 2000] J. Krabbendam and J.-J.Ch. Meyer. Release logics for temporalizing dynamic logic, orthogonalising modal logics. In M. Barringer, M. Fisher, D. Gabbay, and G. Gough, editors, *Advances in Temporal Logic*, pages 21–45. Kluwer Academic Publisher, 2000.
- [Krabbendam and Meyer, 2003] J. Krabbendam and J.-J. Ch. Meyer. Contextual deontic logics. In P. McNamara and H. Prakken, editors, *Norms, Logics and Information Systems*, pages 347–362, Amsterdam, 2003. IOS Press.
- [Kraus *et al.*, 1990] S. Kraus, D. J. Lehmann, and M. Magidor. Nonmonotonic reasoning, preferential models and cumulative logics. *Artificial Intelligence*, 44(1-2):167–207, 1990.
- [Lewis, 1969] D. Lewis. *Convention*. Harvard University Press, Cambridge, MA, 1969.
- [Lindahl and Odelstad, 2000] L. Lindahl and J. Odelstad. An algebraic analysis of normative systems. *Ratio Juris*, 13:261–278, 2000.
- [Lindahl and Odelstad, 2006] L. Lindahl and J. Odelstad. Open and closed intermediaries in normative systems. In T.M. van Engers, editor, *Proceedings of the Nineteenth JURIX Conference on Legal Knowledge and Information Systems (JURIX 2006)*, pages 91–100, 2006.
- [Lindahl and Odelstad, 2008a] L. Lindahl and J. Odelstad. Intermediaries and interventions in normative systems. *Journal of Applied Logic*, 6(2):229–258, 2008.
- [Lindahl and Odelstad, 2008b] L. Lindahl and J. Odelstad. Strata of intervenient concepts in normative systems. In R. van der Meyden and L. van der Torre, editors, *Deontic Logic in Computer Science, 9th International Conference, Proceedings of DEON 2008*, volume 5076 of *LNCS*, pages 203–217. Springer, 2008.
- [Lorini and Longin, 2008] E. Lorini and D. Longin. A logical account of institutions: from acceptances to norms via legislators. In G. Brewka and J. Lang, editors, *International Conference on Principles of Knowledge Representation and Reasoning (KR)*, Sydney, Australia, 16/09/08-19/09/08, pages 38–48. AAAI Press, 2008.

- [Lorini *et al.*, 2009] E. Lorini, D. Longin, B. Gaudou, and A. Herzig. The logic of acceptance: Grounding institutions on agents' attitudes. *Journal of Logic and Computation*, To appear., 2009.
- [Makinson and van der Torre, 2000] D. Makinson and L. van der Torre. Input-output logics. *Journal of Philosophical Logic*, 29(4):383–408, 2000.
- [Makinson, 1999] D. Makinson. On a fundamental problem of deontic logic. In P. McNamara and H. Prakken, editors, *Norms, Logics and Information Systems. New Studies in Deontic Logic and Computer Science*, volume 49 of *Frontiers in Artificial Intelligence and Applications*, pages 29–53. IOS Press, Amsterdam, 1999.
- [Nute, 1987] D. Nute. Defeasible logic. In *Handbook of Logic in Artificial Intelligence and Logic Programming*, volume 3, pages 353–395. Oxford University Press, 1987.
- [Peczenik, 1989] A. Peczenik. *On Law and Reason*. Kluwer, Dordrecht, 1989.
- [Pufendorf, 1688] S. Pufendorf. *De Jure Naturae et Gentium*. Clarendon Press, 1934 edition, 1688.
- [Rawls, 1955] J. Rawls. Two concepts of rules. *The Philosophical Review*, 64(1):3–32, 1955.
- [Ricciardi, 1997] M. Ricciardi. Constitutive rules and institutions. Paper presented at the meeting of the Irish Philosophical Club, Ballymascanlon, February 1997.
- [Ross, 1957] A. Ross. Tû-tû. *Harvard Law Review*, 70:812–825, 1957.
- [Searle, 1969] J. Searle. *Speech Acts. An Essay in the Philosophy of Language*. Cambridge University Press, Cambridge, 1969.
- [Searle, 1995] J. Searle. *The Construction of Social Reality*. Free Press, 1995.
- [Skyrms, 2010] B. Skyrms. *Signals - Evolution, Learning and Information*. Oxford University Press, Oxford, UK, 2010.
- [Tarski, 1944] A. Tarski. The semantic conception of truth and the foundations of semantics. *Philosophy and Phenomenological Research*, 4(3):341–376, 1944.
- [Tarski, 1983] A. Tarski. The establishment of scientific semantics. In *Logic, Semantics, Metamathematics*, pages 401–408. Hackett, Indianapolis, 1983.
- [Von Wright, 1963] G. H. Von Wright. *Norm and Action. A Logical Inquiry*. Routledge, London, 1963.