Logic in the Law: A Concise Overview *

Davide Grossi† and Antonino Rotolo‡

† ILLC - University of Amsterdam
d.grossi@uva.nl

‡CIRSFID - University of Bologna
antonino.rotolo@unibo.it

1 Introduction

The relationship between logic and law has been a troublesome one and it has been object of much philosophical debate in the last century (cf. [26]). Several scholars have denied the usefulness of logical methods in law and legal theory while others have strongly argued in favor of a logic-driven analysis of law and legal reasoning (e.g., [31]). Be it as it may, this latter view has generated decades of interesting work at the interface of law, logic but also philosophy and artificial intelligence, and this work is the object of the present chapter.

H. L. A. Hart, among other philosophers, clarified the roles and functions that systems of norms and normative reasoning play in the law [22]:

Norm recognition and hierarchies: legal systems provide criteria for establishing whether norms belong to them; also, legal systems assign to their norms a different ranking status and organize them in hierarchies;

Norm application: legal systems include criteria to correctly apply their norms to concrete cases;

Norm change: legal systems identify criteria governing their dynamics.

Theoretically speaking, legal logic is a discipline that revolves around all the above three aspects of the law. The present chapter is devoted to a short and light-weighted overview of some significant contributions on those issues. The overview should by no means be considered exhaustive and just briefly considers some key issues selected by the authors.

Outline. The chapter will be structured around four sections each addressing, in a concise way, a particular aspect of legal theory which has been object of analysis by logical methods. The first two sections deal with the study of those basic notions that are most typically associated with legal reasoning, viz.

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deontic notions such as obligation, permission, prohibition, a study that is preliminary for any investigation on all the three issues mentioned above. Section 2 reviews some of the basic ideas that lie at the core of the analysis of so-called *deontic logic* as based on modal logic. Section 3 focuses on those approaches that study norms not as logical formulae bearing truth-values, but rather as primitive and further unanalyzable components of highly structured systems called *normative systems*. Section 4 addresses a characteristic feature of legal reasoning, which goes under the name of *defeasibility* referring to the fact that legal conclusions can often be 'defeated' when new evidence becomes available. Legal defeasibility has been largely studied in the context of judicial and norm-application reasoning, but it is also relevant in regard to the nature of norms and legal systems. Finally, Section 5 deals with the topic of norm change and legal dynamics. The chapter will be briefly recapitulated in Section 6.

2 Deontic logic

To put it with [28, p. 47], deontic logic is "[...] essentially concerned with the representation and analysis of reasoning about a fundamental distinction that arises naturally in law: the distinction between what *ideally* is the case on the one hand, and what *actually* is the case on the other."

Without aiming at providing a comprehensive historical presentation of the rich area of deontic logic—excellent overviews are available, e.g., in [24]—we will here rather point to a few key ideas which, we believe, lie at the core of the field.

2.1 Deontic modalities

At the heart of the development of deontic logic lies the recognition that deontic modalities, e.g., "it is obligatory/permitted that", logically behave like the familiar "it is necessary/possible that ...".

This observation was the stepping stone of Von Wright's paper [67], which has to all effects sparked the modern tradition in deontic logic as based on modal logic. In fact, the observation had already been made by Leibniz, who called these modalities "Iuris Modalia"[36, p. 468], literally, *legal modalities*. Interestingly, Leibniz went on stressing that such modalities can easily be fitted into the canonical square of Aristotelian oppositions:[1]

Obligatory — Obligatory not

Permitted — Permitted not

In this line of thought, Von Wright proposed a formal analysis of the deontic modalities **P** ("it is permitted that ...") and **O** ("it is obligatory that ...") via an axiomatic system conceived as an extension of propositional logic. This system corresponds to the modal logic nowadays known as KD[2] (although not

[1] "Omnes ergo Modalium complicationes et transpositiones et oppositiones, ab Aristotele aliisque in Logicis demonstratae ad haec nostra Iuris Modalia non inutiliter transferri possunt." [36, p. 468]

[2] The reader is referred to [9] for an exposition of the system.

completely[3]) and whose key axiom consisted in what he called the *principle of permission*:

$$\mathbf{P}p \vee \mathbf{P}\neg p \tag{1}$$

which expresses a form of deontic consistency requirement, by ruling out the possibility of situations where no course of action is allowed.

Von Wright also noticed that Formula 1 implies the principle

$$\mathbf{O}p \rightarrow \mathbf{P}p \tag{2}$$

i.e., if p is obligatory then it is permitted[4] although he did not notice their equivalence.

2.2 Semantics of deontic modalities

A semantic analysis of deontic modalities had to wait for the advent, about ten years later, of Kripke semantics. Von Wright could not know at that time that Formulae 1 and 2 are validities of models where $\mathbf{O}$ and $\mathbf{P}$ are interpreted as universal and, respectively, existential quantification along a binary *accessibility* relation, which is assumed to be serial.[5] Kripke himself made this observation in [35], thereby establishing the first semantic analysis of deontic logic.[6]

With modern modal logic notation, let $\mathcal{M} = \langle S, R, \mathcal{V} \rangle$ be a Kripke model with S a non-empty set of states, $R \subseteq S^2$ a serial binary relation on S, and $\mathcal{V} : \mathbf{P} \longrightarrow \wp(S)$ a valuation function from the set of atoms $\mathbf{P}$ to subsets of S. The semantics for modal operators $\mathbf{O}$ and $\mathbf{P}$ is then spelled out as follows:

$$\mathcal{M}, s \models \mathbf{O}\varphi \quad \text{IFF} \quad \forall s' : \text{ IF } sRs' \text{ THEN } \mathcal{M}, s' \models \varphi \tag{3}$$

$$\mathcal{M}, s \models \mathbf{P}\varphi \quad \text{IFF} \quad \exists s' : sRs' \text{ AND } \mathcal{M}, s' \models \varphi \tag{4}$$

where $s \in S$ and φ is a formula from the language of propositional logic extended with monadic operators $\mathbf{P}$ and $\mathbf{O}$.

The intuition behind this semantics is extremely simple: φ is obligatory, respectively permitted, if and only if φ is the case for all *ideal alternatives* to the current one, resp., if and only if there exists some *ideal alternative* to the current one, where φ is the case. Seriality imposes the natural constraint that ideal alternatives always exist.

A serial ideality relation is the trademark of what is now often called standard deontic logic (SDL). It soon became apparent, however, that this semantics was not able to satisfactorily capture a key notion of common deontic reasoning, viz. the notion of contrary-to-duty obligation. These are obligations that are triggered by the violation of other obligations. E.g., "you ought not to kill, but if you kill you ought to do it in self-defense", or "you ought to return your books

[3] In his system, Von Wright assumed a weaker form of axiom K.

[4] The principle had already been observed by Leibniz via analogy with the Aristotelian square of oppositions mentioned above: "omne debitum est justum" [36, p. 468], i.e., everything that is obligatory is permitted (just).

[5] A binary relation R is serial if and only if $\forall x \exists y : xRy$.

[6] To be precise, other scholars beside Kripke had proposed related semantic ideas in those years, in particular Kanger and Hintikka. However, it is Kripke's technical notions and methods that became dominant. For an interesting and well-documented overview of the development of the early semantic analysis of deontic logic, we refer the reader to [66].

to the library on time, but if you do not you ought to pay a fine". Roughly, contrary-to-duty obligations have to do with sub-ideal, or reparatory obligations. That this notion is impossible to capture in SDL was made manifest in the literature by a number of scenarios—often called, with a stretch, paradoxes.[7]

A simple workaround the problem was offered by a semantics proposed in [20], the first articulated paper on the semantics of deontic concepts. The idea is to substitute the serial ideality relation by a total preorder $\succeq$, i.e., a reflexive, transitive and total binary relation, with the following intuitive reading: $s \succeq s'$ means that state s is at least as good/ideal as s'.[8] Now the most ideal states are the maximal of such an order, and sub-ideality can easily be represented by considering the maximals of some subset of states. On this basis, dyadic obligations of the type "it is obligatory that φ under condition ψ" are interpreted as follows:

$$\mathcal{M}, s \models \mathbf{O}(\varphi \mid \psi) \text{ IFF } Max_{\succeq}(||\psi||_{\mathcal{M}}) \subseteq ||\varphi||_{\mathcal{M}} \tag{5}$$

where $||.||_{\mathcal{M}}$ denotes the truth-set function of $\mathcal{M} = \langle S, \succeq, \mathcal{V}\rangle$ and $Max_{\succeq}$ the function extracting the maximals of a given set. A contrary-to-duty obligation will then be represented by taking condition ψ to be the violation of some other obligation.[9] Formula 5 gave rise to a whole line of investigation into the so-called preference-based semantics of deontic logic, which enjoyed considerable attention up till the Nineties[10] and has been recently revived in [63].

Finally, it is worth to mention an alternative, rather simple, approach to the semantics of deontic notions which, in a nutshell, interprets the sentences "φ ought to be the case" as "$\neg\varphi$ necessarily gives rise to an *illegal* situation". This interpretation dates back to Anderson's [4] and Kanger's [32] so-called 'reduction' of deontic logic:

$$\mathcal{M}, s \models \mathbf{O}\varphi \quad \text{IFF} \quad \mathcal{M}, s \models \Box(\neg\varphi \rightarrow \mathsf{V}) \tag{6}$$

[7]One paradigmatic example is the so-called 'gentle murder' paradox:

> "Let us suppose a legal system which forbids all kinds of murder, but which considers murdering violently to be a worse crime than murdering gently. [...] The system then captures its views about murder by means of a number of rules, including these two:
>
> 1. It is obligatory under the law that Smith not murder Jones.
> 2. It is obligatory that, if Smith murders Jones, Smith murders Jones gently." [12, p. 194]

The first obligation is clearly formalizable as $\mathbf{O}\neg m$. What about the second? There are two options: a) $m \rightarrow \mathbf{O}g$; or b) $\mathbf{O}(m \rightarrow g)$. Now assume that m is actually the case. By a) we would conclude that $\mathbf{O}g$, i.e., that it is obligatory to murder gently in the first place, and hence that $\mathbf{O}m$ (assuming that g logically implies m), thereby reaching a contradiction. By b) we could also reach a contradiction by deriving that $\mathbf{P}(\neg m \vee g)$ and hence $\mathbf{P}g$ from which $\mathbf{P}m$ (assuming, again, that g logically implies m). For a more extensive overview of similarly problematic scenarios for SDL we refer the reader to the aforementioned [24] and [5].

[8]Other conditions could obviously be imposed on the relation, typically yielding orderings that are weaker than total preorders.

[9]Interestingly, from the end of the 1960s, the very same idea behind Formula 5 has reappeared in many other branches of philosophical logic like, eminently, conditional logic and doxastic logic. In conditional logic, the expression $\max(||\psi||_{\mathcal{M}}) \subseteq ||\varphi||_{\mathcal{M}}$ has been used to give a semantics for counterfactual conditionals $\psi \Rightarrow \varphi$ [60, 37], and in doxastic logic, to give a semantics for conditional beliefs $\mathbf{B}(\varphi \mid \psi)$.

[10]See [64] for an overview.

where the logic of $\Box$ ranges, in the literature, between modal systems T and $\mathsf{S5}$,[11] and where V represents a designated 'violation' constant. As recently shown in [63], this reductionist approach to deontic logic can be straightforwardly related to the semantics based on ideality ordering which we described above.

2.3 Beyond obligation and permission

Obligation and permission are only two of the rich family of concepts that play a role in normative, and in particular legal, reasoning. A first rich source of analysis of related concepts (eminently the notions of right and power) can be found in Hohfeld's [25], an influential paper on the analytical philosophy of law. The sort of analysis proposed in [25] is exemplified by what are sometimes called the Hohfeldian squares, of which these are the two main examples:

$$\begin{tikzcd}
\text{Right} \arrow[r, dash] \arrow[d, dash] & \text{Obligation} \arrow[d, dash] \\
\text{No-right} \arrow[r, dash] & \text{Privilege}
\end{tikzcd}
\qquad
\begin{tikzcd}
\text{Power} \arrow[r, dash] \arrow[d, dash] & \text{Liability} \arrow[d, dash] \\
\text{Disability} \arrow[r, dash] & \text{Immunity}
\end{tikzcd}$$

Right and obligation (or duty, in Hohfeld's terminology) are viewed as correlatives: if i (the bearer) has a right against j (the counterparty) that φ is brought about, then j has the obligation toward i to bring about φ. A privilege is the opposite of an obligation, e.g., j is not obliged toward i to bring about φ. Similarly is no-right the opposite of right and the correlative of privilege. After the advent of deontic logic, it became clear that these notions could be object of formal analysis. In fact, Hohfeld's line of research has later been systematically pursued within logic in work devoted to the analysis of so-called *directed obligations*, viz. obligations where bearers and counterparties are made explicit in a specially designed deontic logic (cf. [23]), and by what has come to be known as the Kanger-Lindahl theory of normative positions, which has developed into a rich blend of modal deontic and action logics able to formalize a complex array of deontic and legal concepts.[12]

While the formal analysis of the first square could build on deontic logic as the underlying framework for a logic of obligation, a formal analysis of the second square appeared more problematic. The quest for such analysis was programmatically set by Jones and Sergot in [29], a paper that sparked an interesting line of research at the interface of logic, philosophy and artificial intelligence in the last fifteen years.[13]

The issue addressed by [29] consists in the formal characterization of a notion of *legal power* as involved in sentences such as "the president has the power to declare a state of emergency". This notion of power is viewed as grounded in the so-called constitutive rules, viz. legal rules such as "18 years of age counts as age of majority" or "the president's signature counts as the enactment of the bill". For instance, the latter rule establishes that the president has the power to enact a legislative bill. As extensively argued for instance in [58], these rules—often called counts-as conditionals—represent the basic brick of complex institutions such as legal systems, and [29] developed a first logical analysis of them.

[11] Again, we refer to [9] for a presentation of these logics.
[12] See [33] for an early exposition and [59] for a recent version of the theory.
[13] See [18] for a recent comprehensive overview of this field of research.

3 Normative systems

Another influential formal account of deontic notions, complementary to the (modal) logic-based approaches we discussed in the previous section, is the one sparked by Alchourrón and Bulygin in [2]. The key feature of this approach is to study norms—viewed as dyadic constructs connecting a fact to a deontic consequence (e.g. Formula 5)—not as formulae in some logical language, but rather as primitive ordered pairs $\langle \mathit{condition}, \mathit{consequence} \rangle$. A large number of such pairs would constitute an interconnected system called a *normative system*.

Viewed as parts of a bigger system norms are therefore considered to be uninterpretable if taken in isolation—unlike in logical semantics—and they acquire meaning only by relating to other norms in the system. The focus falls then on the problem of normative reasoning and its most characteristic features, such as: defeasibility, to which we will come back in detail in Section 4; the validity of closure principles (e.g., *nullum crimen sine lege*[14]); the problem of handling legal gaps.

The basic idea behind normative systems goes hand in hand with the thesis according to which norms do not bear truth-values, and hence that deontic logics do not actually deal with norms, but rather with normative propositions, i.e., statements to the effect that certain norms exist. For instance, in this view, $\mathbf{O}\varphi$ would actually mean something like "there exists a norm commanding φ".[15]

In what follows we sketch, very briefly, the basic ideas behind two of the approaches that in recent years have taken up and developed the normative systems approach to the analysis of norms: input/output logics and normative systems algebras.

3.1 Input/output logic

Input/output logics (henceforth IOL) are a formalism introduced in [42] that has been applied to the study of normative systems in a long series of papers (e.g. [10]) by viewing them as rule-based process of manipulation of inputs (factual premisses) into outputs (normative conclusions).

The key idea behind the application of IOL to the analysis normative systems consists in representing conditional norms simply as ordered pairs (a, b) where a represents the antecedent of the rule, and b its consequent: "if a then b" where a has factual content and b normative content, viz. an obligation or a permission. Typically, both a and b are taken to be formulae from propositional logic. Each set of such ordered pairs can be seen as an inferential mechanism which, given an input, determines an output based on those connections.

Various definitions can be given of how to produce the output on the basis of a set of pairs, and all consist in ways of closing the given set of pairs by adding new pairs in accordance to some principles, of which we give two very simple

[14] No crime without law, that is, everything that is not explicitly prohibited should be considered as permitted.

[15] The problem of whether norms bear or not truth values is an old one in philosophy, and was put forth in modern times by [30]. The significance of the problem has recently been reemphasized in [19], and a new approach to the problem emerged from the view of norms as 'dynamic' operators—speech acts—modifying ideality orders. We will briefly come back to this latter point in Section 5.

examples:

$$SI : \frac{(a, b)}{(a \wedge c, b)} \qquad CT : \frac{(a, b), (a \wedge b, c)}{(a, c)} \tag{7}$$

where SI stands for strengthening of the input—essentially an antecedent strengthening property—and CT stands for cumulative transitivity. Formally, given a set $NORM$ of pairs, a closure operation C defined in terms of some of the above principles, and a set of facts A, the output of $NORM$ given C and a set of input formulae I is:

$$out_C(NORM, A) \quad = \quad \{b \mid (a, b) \in C(NORM) \text{ and } s \in A\} \tag{8}$$

Intuitively, $NORM$ represent the norms of a normative system and C the principles according to which the system makes the norms interact with one another. As the reader might have already noticed, this represents a very high-level abstraction of the workings of a normative system. Depending on the (many) ways the output operation is defined, IOL can be used to capture vary different principles for reasoning with norms (among which defeasibility, Section 4). This modeling freedom brought IOL to be applied not so much to the study and analysis of normative reasoning in *actual* legal systems, but rather to the specification of *artificial* normative systems in the field of artificial intelligence (see the aforementioned [10]).

3.2 Algebras of normative systems

With [38], Lindahl and Odelstad advocate an algebraic analysis of normative systems. The approach is very close in spirit to the one, discussed above, of IOL. However, the formal machinery deployed is not based on logic and hinges on several algebraic and order-theoretical notions. In this section we provide just a brief sketch of the basic technical ideas underpinning the framework.

According to this approach norms can be seen—exactly as in IOL—as simple pairs $\langle a, b \rangle$ connecting (factual) conditions to (normative) consequences. Both conditions a and consequences b are taken to be elements of a set X upon which a Boolean algebra $\langle X, \sqcap, -, \bot \rangle$ is defined. Within such a structure, the normative relation between condition a and consequence b is given by extending the preorder yielded by the algebra.[16] The idea is that while the preorder—let us call it $\preceq$—represents some form of logical implication, normative systems add on the top of it the possibility of drawing more conclusions by some form of 'legal' implication—let us call it ρ. In other words, each normative system introduces, by stipulation, a consequence relation which is stronger than the logical one: $\preceq \subseteq \rho$. The intuition is that, for instance, the fact that being obliged to pay taxes follows from having a paid job is not a matter of logic, but a matter of stipulation[17].

[16] A preorder can always be associated to a given Boolean algebra in the following way:

$$a \preceq b \qquad \text{IFF} \qquad a \sqcap b = a. \tag{9}$$

[17] As is well-known, the idea that legal effects do not follow from norms by logic but, rather, by stipulation was notably defended in legal theory by [34].

Therefore, in Lindahl and Odelstad's view normative systems can be studied as Boolean algebras supplemented by a binary relation ρ. This is, in a nutshell, the key idea behind the approach. Space limitation prevents us to provide more details. It should be mentioned, however, that [38] was followed by a number of papers developing an extensive theory of normative systems on the ground of the simple intuition we have sketched above.[18]

4 Defeasibility in legal reasoning

One key idea of most logical accounts of the law is that legal reasoning is defeasible, namely, that we may have reasons to abandon certain legal conclusions even though there was no apparent mistake in previously supporting them [57]. In legal theory, H.L.A. Hart was the first who illustrated this idea by saying, for instance, that "there are positive conditions required for the existence of a valid contract" but there are reasons that can defeat that existence claim, "even though all these conditions are satisfied" [21, p. 152]. The concept of defeasibility may have in the law different connotations.

4.1 Meanings of 'defeasibility' in the law

Consider art. 2051 of the Italian civil code: "A person is liable for damage caused by things in his custody except where he shows evidence of a fortuitous case". This legal provision states that the fault is not required to show the liability of the receiver for damage caused by things in safekeeping, thus highlighting the fact that the applicability conditions of legal norms include both conditions that should be proved and conditions that should not be refuted (in this case, the fact that the receiver is at fault) [56].

Conditions of the latter type can be explicit, like in the above provision, but are most often implicit. In general, the fact is that the statement of a norm can never mention all the relevant issues that might possibly be of relevance for its application, and in particular all its possible exceptions. This 'openness' to possible exceptions is a characteristic feature of legal norms and is known to be a peculiar aspect of legal *defeasibility*.

Defeasibility in legal norms breaks down, roughly, into the following issues:

Conflicts. Norms can conflict, namely, they may lead to incompatible legal effects. Conceptually, conflicts can be of different types, according to whether two conflicting norms

1. are such that one is an exception to the other (i.e., one is more specific than the other); this type of conflict can be solved using the principle *lex specialis*, which gives priority to the more specific norms (i.e., the exceptions);
2. have a different ranking status; this type of conflict can be solved using the principle *lex superior*, which gives priority to the norm from the higher authority;

[18] An interesting recent contribution is, for instance, [39]

3. have been enacted at different times; this type of conflict can be solved using the principle *lex posterior*, which gives priority to the norm enacted later.

Exclusionary norms. Some norms provide one way to explicitly undercut other norms, namely, to make them inapplicable.

Contributory reasons or factors. It is not always possible to formulate precise norms, even defeasible ones, for aggregating the factors relevant for resolving a legal issue. For example: "The educational value of a work needs to be taken into consideration when evaluating whether the work is covered by the copyright doctrine of fair use".

There are however more general reasons why legal reasoning should be viewed as defeasible. In fact, not all legal norms distinguish different types of applicability conditions (what should be proved and what should not be refuted), or not all norms admit exceptions or can be defeated. Independently of this, one may argue that legal reasoning is part of human cognition, which is defeasible [44] or that, even when norms seem to support indisputable conclusions, they are used in legal disputes or, more generally, in legal argumentative settings where arguments and counter-arguments dialectically interact.

When looking at the law through an argumentative lens, we may distinguish inference-based defeasibility, process-based defeasibility, and theory-based defeasibility [50].

Inference-based defeasibility covers the fact that legal conclusions, though correctly supported by certain pieces of information, cannot be derived when the knowledge base including those information is expanded with further pieces of information.

Process-based defeasibility addresses the dynamic aspects of defeasible reasoning. As for legal reasoning, a crucial observation here is that it often proceeds according to the norms of legal procedures, such as those regulating the allocation of the burden of proof.

Theory-based defeasibility regards the evaluation and the choice of theories which explain and systematize the available legal input information (such as a set of precedents): when a better theory becomes available, inferior theories are to be abandoned.

The remainder of this section briefly discusses aspects of the first two types of defeasibility. As for the the third type (theory-based defeasibility), the interested reader can still find a good primer in [50, sec. 4].

4.2 Defeasibility and argumentation layers in the law

Defeasible reasoning has been largely investigated in philosophy, logic, and AI by usually working on the concept of *inference-based defeasibility* [41]. In this sense, defeasibility is formally interpreted within non-monotonic logics, namely, in logics whose underlying consequence relation does *not* enjoy monotonicity, i.e., that conclusions do not decrease if more knowledge is added. Since non-monotonicity means that a logic lacks a property, its positive interpretation is open to many options. In regard to modeling legal reasoning, since the Nineties the most preferred one (especially in the AI&Law community) has been to develop argumentation systems (see, e.g., [14, 40, 47, 8, 15][19].

The advantage of this approach is that it intuitively captures the dialectal nature of legal reasoning by clearly considering its different layers. In particular, this approach at least distinguishes a logical layer, a dialectical layer, and a procedural layer of legal arguments [49, 52].

Logical layer The logical layer deals with the underlying language that is used to build legal arguments. Many languages and reasoning methods can be used for this purpose, such as deduction, induction, abduction, analogy, and case-based reasoning[20]. If the underlying language refers to logic $\mathbf{L}$, arguments can roughly correspond to proofs in $\mathbf{L}$ [49]. It may be argued that most (legal) argumentation systems are based on a *monotonic* consequence relation, since each single argument cannot be revised but can only be invalidated by *other* arguments (or better, counter-arguments) [52]: it is the exchange of arguments and counter-arguments that make the system non-monotonic. However, this is not strictly required: when the underlying logic is itself non-monotonic, an argumentation system can be simply seen as an alternative way to compute conclusions in that non-monotonic logic [16][21].

Suppose we resort to a rule-based logical system where rules have the form $\varphi_1, \ldots, \varphi_n \Rightarrow \varphi$ and represent defeasible legal norms. An argument for a legal conclusion φ can typically have a tree-structure, where nodes correspond to literals and arcs correspond to the rules used to obtain these literals; hence, the root corresponds to φ, the leaf nodes to the primitive premisses, and for every node corresponding to any literal ψ, if its children are $\psi_1, \ldots, \psi_n$, then there is a rule whose antecedents are these literals [16].

Argumentation systems, however, do not need in general to specify the internal structure of their arguments [11], so this assumption applies, too, to the legal domain. In this perspective, any (legal) argumentation system $\mathcal{A}$ is a structure $(A, \rightsquigarrow)$, where A is a non-empty set of arguments and $\rightsquigarrow$ is binary attack relation on A: for any pair or arguments a and b in A, $a \rightsquigarrow b$ means that a attacks b. This leads us to discuss the dialectical layer.

Dialectical layer The dialectical layer addresses many interesting issues, such as when legal arguments conflict, how they can be compared and what legal arguments and conclusions can be justified.

Different types of attacks and defeat relations can apply to legal arguments. [44]'s original distinction between *rebutting* and *undercutting* is almost universally accepted in the legal-argumentation literature [49, 50]. An argument A_1 rebuts an argument A_2 when the conclusion of A_1 is equivalent to the negation of the conclusion of A_2. The rebutting relation is symmetric. For example, if arguments are built using rules representing legal norms (regulating, for example,

[19]Although it does not consider the most recent proposals, a still good introductory discussion can be found in [49]

[20]The application of these reasoning methods in the law have been studied by legal logicians, but space reasons prevent us to handle here this discussion. See [57].

[21]If we embed within this language any deontic operators, we will obtain a way to deal with the defeasibility of the corresponding deontic concepts [43]. In general, various forms of interaction can be found among defeasibility, deontic concepts and normative systems. See [57].

smoking in public spaces), a conflict of this type at least corresponds to a clash between the conclusions obtained from two norms (for example, one prohibiting and another permitting to smoke). The undercutting is when an argument challenges a rule of inference of another argument. This attack relation is not symmetric and occurs when an argument A_1 supporting the conclusion φ has some ground ψ but another argument A_2 states that ψ is not a proper ground for φ. To put it very simple, if one builds an argument A_1 for φ using the rules $\Rightarrow \psi$ and $\psi \Rightarrow \varphi$ but we contend that ψ is the case, then we undercut A_1.

Conflicts between legal arguments can be solved using specific legal-domain dependent priority criteria such as, as we said, *lex specialis*, *lex superior*, and *lex posterior*. However, such criteria can conflict, too, so some researchers argued that they must be defeasible [48, 45].

In general, assessing conflicting legal arguments cannot work if we only examine single pairs of arguments. In fact, we need to consider all the arguments to establish what legal conclusions win and are justified in a legal dispute. Argumentation theory usually distinguishes among *justified*, *defensible* and *overruled* arguments. Justified arguments are those which basically survive from all attacks, the defensible ones leave the dispute undecided, and the overruled ones are those defeated by a justified argument [52]. Doing so, we may have to capture interesting complex argumentative patterns. For instance, consider this argumentation system:

$$(A = \{A_1, A_2, A_3\}, \rightsquigarrow = \{\langle A_1, A_2\rangle, \langle A_2, A_1\rangle, \langle A_3, A_2\rangle\})$$

The argument A_1 is attacked and defeated by the argument A_2 but it may be reinstated when a third argument A_3 attacking A_2 comes into play [52]. This is an example of a reasoning pattern known in argumentation theory as *reinstatement*, which is relevant, for instance, in legal evidential reasoning: suppose Henry was killed yesterday and John was charged with that crime. Tom argues that John did not kill yesterday Henry, but Nino testifies that John indeed killed him. Tom original argument can be reinstated by another testimony showing that that Nino was drunk yesterday.

Another interesting pattern regards the so-called floating conclusions [27]. Consider the following two arguments (represented as as chains of rules):

$$\begin{array}{llll} A_1 & \mathit{testimonyA} & \Rightarrow \mathit{JohnShootHenry} & \Rightarrow \mathit{guilty} \\ A_2 & \mathit{testimonyB} & \Rightarrow \mathit{JohnPoisonHenry} & \Rightarrow \mathit{guilty} \end{array} \tag{10}$$

The two arguments lead to the same conclusion but one sub-argument of A_1 attacks one sub-argument of A_2 and vice versa (the fact that John shot Henry excludes that John poisoned Henry and vice versa). One may say that John is anyway guilty, whatever argument we may prefer, but we can also argue that the two testimonies undermine each other, so no conclusion could be obtained.

Procedural layer The procedural layer considers the ways through which conclusions are dynamically reached in legal disputes. Indeed, disputes can be reconstructed in the form of dialogues, namely of players' dialectical moves [14, 46]. Legal disputes in turn are regulated by procedural rules stating what dialogue moves (claiming, challenging, conceding, etc.) are possible, when they are legal, what effects the players get from them, and under what conditions a dispute terminates [14, 40] (in general, see [65])[22].

A basic and fundamental question of the procedural layer regards how to govern and allocate the burden of proof [46]. For example, basic dialogue protocols of 2-player civil disputes are defined on account of the requirement that the plaintiff begins the dispute with his claim and has to propose, to win, at least one justified argument which support such a claim. The burden of the defendant is not in principle the same, as it may be sufficient in most cases for her to oppose to the plaintiff argument moves that are only defensible counter-arguments. The concept of legal burden of proof is very complex and its logical treatment is difficult: the interested reader can refer to [51]. Even more complex is to handle the interplay between the dialectical and the procedural layers [46]. To appreciate this, consider the example on floating conclusions of Formula 10. Here, players, if dynamically modeled at the procedural layer would certainly postpone their judgment and subsequently challenge both the testimonies and test their credibility [50].

5 Legal dynamics

One peculiar feature of the law is that it necessarily takes the form of a dynamic normative system [34, 22]. Despite the importance of norm-change mechanisms, the logical investigation of legal dynamics is still much underdeveloped. However, recent contributions exist and this section is devoted to a brief sketch of this rapidly evolving literature.

5.1 AGM-based approaches

In the Eighties a promising research effort was devoted by C. E. Alchourrón, P. Gärdenfors and D. Makinson to develop a logical model (AGM) for modeling norm change. As is well-known, the AGM framework distinguishes three types of change operation over theories. Contraction is an operation that removes a specified sentence φ from a given theory Γ (a logically closed set of sentences) in such a way as Γ is set aside in favor of another theory Γ^-_φ which is a subset of Γ not containing φ. Expansion operation adds a given sentence φ to Γ so that the resulting theory Γ^+_φ is the smallest logically closed set that contains both Γ and φ. Revision operation adds φ to Γ but it is ensured that the resulting theory Γ^*_φ be consistent [1]. Alchourrón, Gärdenfors and Makinson argued that, when Γ is a code of legal norms, contraction corresponds to norm derogation (norm removal) and revision to norm amendment.

AGM framework has the advantage of being very abstract but works with theories consisting of simple logical assertions. For this reason, it is perhaps suitable to capture the dynamics of obligations and permissions, not of legal norms. In fact, it is essential to distinguish norms from obligations and permissions [13, 17]: the latter ones are just possible effects of the application of norms and their dynamics do not necessarily require to remove or revise norms, but correspond in most cases to instances of the notion of *norm defeasibility* [17]. Very recently, some research has been carried out to reframe AGM ideas within

[22] The idea that justice depends on formal procedures governing public deliberation and dialogues has been defended, among others, in [54, 53] and in the law in [3].

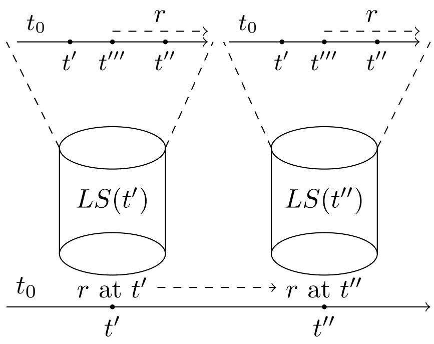

Figure 1: Legal System at t' and t''

rule-based logical systems, which take this distinction into account [61, 55]. However, also these attempts suffer from some drawbacks, as they fail to handle the following aspects of legal norm change:

1. the law usually regulate its own changes by setting specific norms whose peculiar objective is to change the system by stating what and how other existing norms should be modified;

2. since legal modifications are derived from these peculiar norms, they can be in conflict and so are defeasible;

3. legal norms are qualified by temporal properties, such as the time when the norm comes into existence and belongs to the legal system, the time when the norm is in force, the time when the norm produces legal effects, and the time when the normative effects hold.

Hence, legal dynamics can be hardly modeled without considering defeasibility and temporal reasoning. Some recent works (see, e.g., [17]) have attempted to address these research issues. All norms are qualified by the above mentioned different temporal parameters and the modifying norms are represented as defeasible meta-rules, i.e., rules where the conclusions are temporalized rules.

If $t_0, t_1, \ldots, t_j$ are points in time, the dynamics of a legal system LS are captured by a time-series $LS(t_0), LS(t_1), \ldots, LS(t_j)$ of its versions. Each version of LS is called a *norm repository*. The passage from one repository to another is effected by legal modifications or simply by temporal persistence. This model is suitable for modeling complex modifications such as retroactive changes, i.e., changes that affect the legal system with respect to legal effects which were also obtained before the legal change was done. The dynamics of norm change and retroactivity need to introduce another time-line within each version of LS (the time-line placed on top of each repository in Figure 1). Clearly, retroactivity does not imply that we can really change the past: this is "physically" impossible. Rather, we need to set a mechanism through which we are able to reason on the legal system from the viewpoint of its current version but *as if* it were revised in the past: when we change some $LS(i)$ retroactively, this does not mean that we modify some $LS(k)$, $k < i$, but that we move back from the perspective of $LS(i)$. Hence, we can "travel" to the past along this inner time-line, i.e., from the viewpoint of the current version of LS where we modify norms.

Figure 1 shows a case where the legal system LS and its norm r persist from time t' to time t''; however, such a norm r is in force in LS (it can potentially have effects) from time t''' (which is between t' and t'') onwards.

5.2 Dynamic logic approaches

Inspired by recent theoretical and technical developments in the logical study of dynamics—especially the dynamics of informational attitudes such as knowledge and belief[23]—some scholars have proposed models for the 'dynamification' of several kinds of deontic logics.

At the heart of these approaches lies the notion of structure transformation. Let us for instance go back to the semantic analysis of obligation that we gave in Section 2.2 which was based on an ideality ordering. This semantics lends itself easily to a view of obligation dynamics based on ways of manipulating that ideality ordering. To make a simple example, following [63], the enactment of a command that φ be the case could be rendered by the modification of that ideality ordering in such a way that all φ-states are ranked as more ideal than all $\neg\varphi$-states. The upshot is the modeling of different forms of norm dynamics in terms of different operations on their semantic structures. Other recent contributions along these lines, although based on different structures, are for instance [7, 6].

It is finally worth observing that the approaches based on dynamic logic offer a perspective which is in a way complementary to the one described in Section 5.1. Unlike the approaches above, they are—at the present state-of-the-art—blind to much of the fine-grained temporal structure of norm change. At the same time, however, they have the advantage of maintaining a clear link with the underlying logical semantics of deontic notions. How the two perspectives can be technically bridged is very much an open issue.

6 Conclusions

Going back to Hart's subdivision by which we opened the chapter, we can see how the lines of research we have touched upon in the above sections cover the different functions of normative reasoning in the law: the theory of normative systems (Section 3) and of defeasibility (Section 4) are attempts to address the first two functions concerning the structure and hierarchies of norms and, respectively, their conditions of application; the theory of legal dynamics (Section 5) provides a way of understanding norm change; finally, deontic logic, can be viewed as a transversal endeavor towards the understanding of the deep fine-grained structure of normative notions as they are presupposed by the above attempts.

We find it worth concluding by pointing the interested reader to those which we consider the main events and forums in the field, and that can be an excellent source of further information, especially on on-going researches. These are:

[23] See [62] for a recent comprehensive overview

the biannual DEON[24] and ICAIL[25], the annual JURIX[26], and the Journal of Artificial Intelligence and Law[27].

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